FIITJEE

JEE (Advanced)-2018

PART TEST – I

Paper 2

ALL INDIA TEST SERIES

Time Allotted: 3 Hours **Maximum Marks: 231**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
4. Each part is further divided into three sections: **Section-A, Section-C & Section-D**.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with black pen for each character of your Enrolment No. and write your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

1. ***Section-A (01 – 03, 24 – 26, 47 – 49)*** *contains 9 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

   Section-A (04 – 08, 27 – 31, 50 – 54) *contains 15 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer.*

   Partial Marks **+1** for each correct option provided no incorrect options is selected.

   Section-A (09 – 10, 32 – 33, 55 – 56) *contains 3 paragraphs. Based upon paragraph, 2 multiple choice questions have to be answered. Each question has* ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer. There is no negative marking.*

2. ***Section-C (11 – 20, 34 – 43, 57 – 66)*** *contains 30 Numerical based questions with answer as numerical value from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

3. ***Section-D (21 – 23, 44 – 46, 67 – 69)*** *contains 9 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Name of the Candidate	
Enrolment No.	

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's constant	h	=	6.625×10^{-34} J·s
		=	6.625×10^{-27} erg·s
1 Faraday		=	96500 coulomb
1 calorie		=	4.2 joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

PART – I (Physics), PART – II (Chemistry), PART – III (Mathematics):

(SECTION – D)

For questions **21 to 23, 44 to 46, 67 to 69.**
Numerical answer type questions with answer XXXXX. XX

If answer is 348.4 / 251.37 / 213

Correct Method :

		3	4	8	.	4	0
		2	5	1	.	3	7
		2	1	3	.	0	0

Wrong Method :

	3	4	8		.	4	
3	4	8			.		4
		3	4	8	.		4
	3		4	8	.	4	
	2		5	1	.	3	7
		2	1	3	.		
		2	1	3	.	0	
		2	1	3	.		0

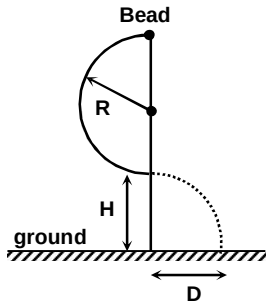
Physics

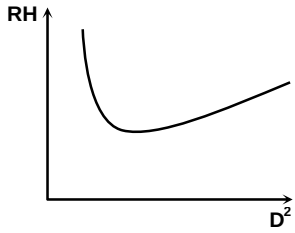
PART – I

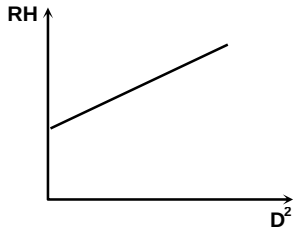
SECTION – A
(One Options Correct Type)

This section contains **3 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A semicircular wire of radius R is oriented vertically. A small bead is released from rest from the top of the wire. It slides without friction under the influence of gravity to the bottom, where it then leaves the wire horizontally and falls distance H to the ground. The bead lands a horizontal distance D away from where it was launched. Which of the following is correct graph of RH versus D^2?



(A)


(B)


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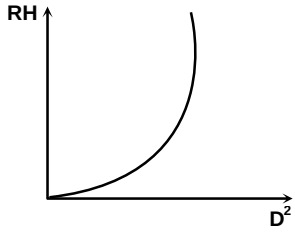
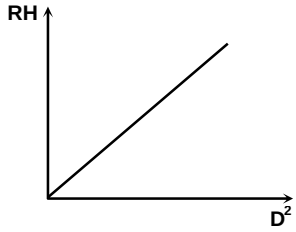
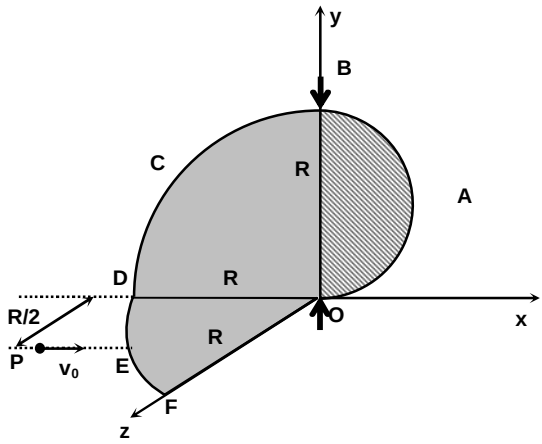


(D)


Space for Rough work

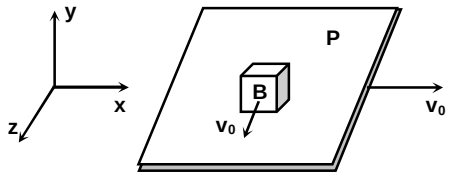
2. A folded plate OABCDEFO made of materials such that part OABO (say part(i)), BCDOB (say part(ii)) and DEFOD (say part(iii)) has mass 4m, m and m respectively. Part (i) is a uniform semicircular plate of radius R/2 and is on the xy plane. Part (ii) and (iii) each is a uniform quarter circular plate of radius R on xy and xz plane respectively. The whole system is free to rotate about y-axis. A particle P of mass m moving with velocity v_0 hits to a point located at the circumference of the part (iii) and sticks to it. The point is at a distance R/2 from x-axis as shown in the figure. Angular velocity of the combined system just after the collision is



(A) $\frac{v_0}{4R}$ (B) $\frac{2v_0}{5R}$

(C) $\frac{3v_0}{5R}$ (D) $\frac{v_0}{2R}$

3. A very large plank P of some unknown mass is being moved with velocity $v_0\hat{i}$ under application of an external force (not shown in figure). Simultaneously a block B of mass m placed on the plank is also moving with velocity $v_0\hat{k}$. All these velocities are with respect to ground frame and at t = 0. Coefficient of friction between the plank and the block is μ. Choose the correct option(s).



(A) Kinetic friction force acting on the block, at t = 0 is $-\mu mg\hat{k}$.

(B) At t = 0, power developed (with respect to ground frame) by kinetic friction force on the block B is $-\frac{\mu mgv_0}{\sqrt{2}}$

(C) At t = 0, power developed (with respect to ground frame) by kinetic friction force on the block B is $-\mu mg\sqrt{2}v_0$

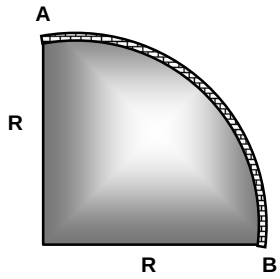
(D) At t = 0, heat dissipation per sec in the system is $2\sqrt{2}\,\mu mgv_0$.

Space for Rough work

(One or More than one correct type)

This section contains **FIVE** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

4. A rope AB of linear mass density λ is placed on a quarter vertical fixed disc of radius R as shown in the figure. The surface between the disc and rope is rough such that the rope is just in equilibrium. Gravitational acceleration is g. Choose the correct option(s).



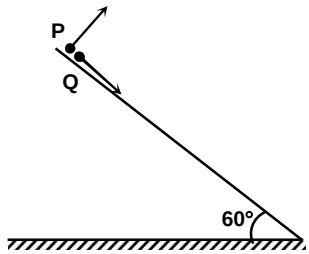
(A) Coefficient of static friction between rope and disc is $\mu = 1$.

(B) Coefficient of static friction between rope and disc is $\mu = \frac{1}{\sqrt{2}}$.

(C) Maximum tension in the rope is at the top most point A of the rope.

(D) Maximum tension in the rope is $\lambda Rg(\sqrt{2}-1)$.

5. A particle P is projected at t = 0 from a point on the surface of a smooth inclined plane as shown in the figure simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after t = 4 seconds. Then choose the correct Option(s).



(A) Trajectory of particle P in the frame of Q is parabola during the flight of particle P.

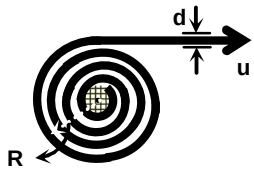
(B) Speed of projection of P is 20 m/s.

(C) Relative velocity of Particle P in the frame of Q changes linearly with time during the flight of P.

(D) Acceleration of particle P in the frame of Q is zero during the flight of P.

Space for Rough work

6. A disc is free to rotate about an axis passing through its centre and perpendicular to its plane. The moment of inertia of the disc about its rotation axis is I. A light ribbon is tightly wrapped over it in multiple layers. The end of the ribbon is pulled out at a constant speed of u. Let the radius of the ribboned disc be R at any time and thickness of the ribbon be d(<<R).



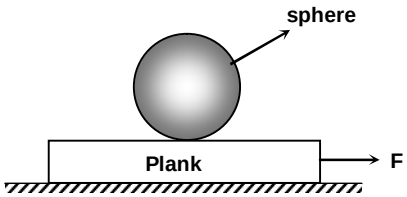
(A) Angular acceleration of the disc at any instant is proportional to $\frac{1}{R^3}$, where R is radius of the ribboned disc.

(B) Angular acceleration of the disc at any instant is proportional to $\frac{1}{R^2}$, where R is the radius of the ribboned disc.

(C) Force required to pull the ribbon at constant speed u is $F = \frac{Iu^2d}{2\pi R^4}$.

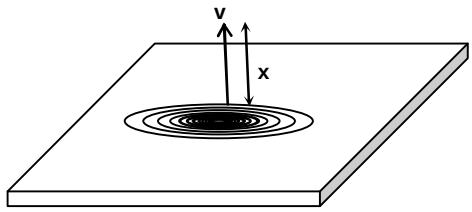
(D) Force required to pull the ribbon at constant speed u is $F = \frac{Iu^2d}{\pi R^4}$.

7. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. The plank is pulled to right by a constant force F. If the sphere does not slip over the plank, then



(A) Both have the same acceleration.
(B) Acceleration of the center of sphere is less than that of the plank.
(C) Work done by friction acting on the sphere is equal to its total kinetic energy.
(D) Total kinetic energy of the system is equal to work done by force F.

8. A uniform rope of liner mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up by an external agent with constant vertical velocity v. Choose the correct option(s).



(A) Power developed by external agent as a function of x is $P = \lambda xgv$
(B) Power developed by external agent as a function of x is $P = (\lambda v^2 + \lambda xg)v$
(C) Energy lost during the complete lift of the rope is zero.
(D) Energy lost during the complete lift of the rope is $\frac{\lambda \ell v^2}{2}$.

Space for Rough work

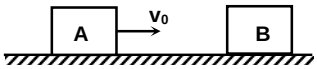
(Paragraph Type)

This section contains **ONE** paragraph. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 9 and 10

Graphical solution of a two body head on collision

A block A of mass m moving with a uniform velocity v_0 strikes another identical block B kept at rest on a horizontal smooth surface as shown in the figure-(i). We can conserve linear momentum.



Smooth horizontal surface

figure-(i)

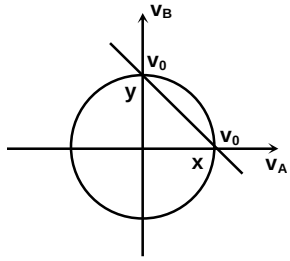


figure-(ii)

So, $mv_0 = mv_A + mv_B$ (v_A and v_B are the velocities of the blocks after collision)

$\therefore \quad v_0 = v_A + v_B$ …(i)

If the collision is perfectly elastic

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$$

$\Rightarrow \quad v_0^2 = v_A^2 + v_B^2$ …(ii)

Both the above equations (i) and (ii) are plotted on $v_A - v_B$ plane as shown in figure (ii). This plot can be used to find the unknowns v_A and v_B.

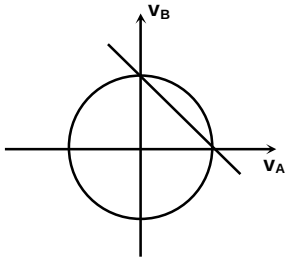
For example, the solution of the situation in figure (i) is $v_A = 0$, $v_B = v_0$ (point y in the plot)

Because $v_A = v_0$, $v_B = 0$ (point x in the plot) is not physically possible.

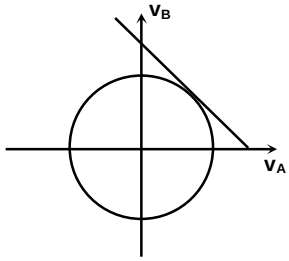
Now answer the following questions based on the above comprehension.

9. If the collision is perfectly inelastic, then the $v_A - v_B$ plot is

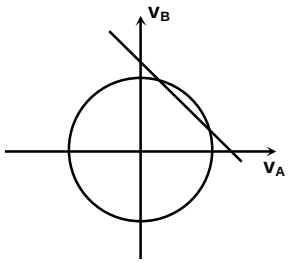
(A)



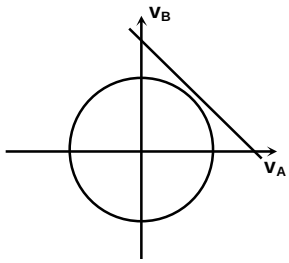
(B)



(C)

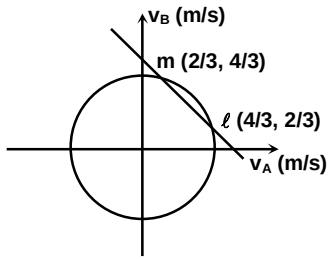


(D)



Space for Rough work

10. In a situation, block A is moving with velocity 2 m/s and strikes another identical block B kept at rest. The $v_A - v_B$ plot for the situation is shown. m and ℓ are the intersection points whose v_A, v_B coordinates are given in the figure. The coefficient of restitution of the collision is
(A) 1/2
(B) 1/3
(C) 1
(D) Collision not possible

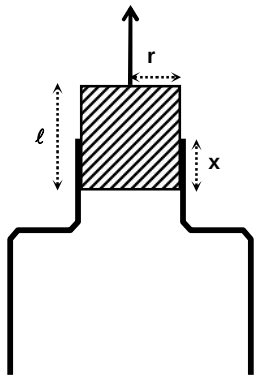


SECTION – C
(Single digit integer type)

This section contains **TEN** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

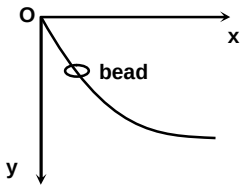
11. A table has a heavy circular top of radius 1 m and mass 2 kg. It has four light legs of length 1 m fixed symmetrically on its circumference. Find the maximum mass (in kg) which may be placed anywhere on this table without toppling it. (take $\sqrt{2} = 1.4$)

12. A cylindrical massless cork of length ℓ = 10 cm and radius r = 3 cm is slowly extracted from the neck of a bottom opened bottle. If the normal pressure between the bottle and un-extracted part of the cork at any instant is constant and equal to $P = \frac{10^5}{\pi}$ N/m^2, find the work done (in Joule) in extracting it completely. The coefficient of friction between the cork and bottle is μ = 0.3.

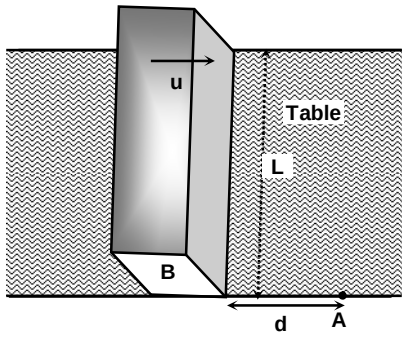


Space for Rough work

13. A bead under the influence of gravity, sides down a frictionless wire whose y coordinate is changing with x co-ordinate as shown in the figure. Assume that at position O the wire is vertical and the bead passes this point with a given speed v_0 downward. If the shape of the wire is such that the vertical component of velocity remains v_0 at all time, find (a + b + c) in the shape function of wire given by $y = \frac{(agv_0x)^{\frac{b}{c}}}{2g}$, where g is gravitational acceleration.



14. A heavy block B is sliding with constant velocity u = 5 m/s on a horizontal table. The width of the block is L = 4 m. There is an insect A at distance d = 3 m from the block as shown in the figure. The insect wants to cross to the opposite side of the table. It begins to crawl at a constant velocity v at the instant shown in the figure. Find the least value of v (in m/s) for which the insect can cross to the other side without getting hit by the block.



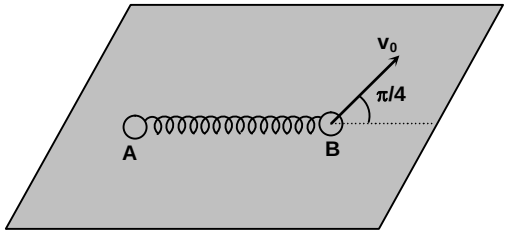
(Top view, slightly skewed)

15. A body is projected vertically upwards with a velocity u = 5 m/s. After time t another body is projected vertically upward from the same point with a velocity v = 3 m/s. If they meet in minimum time duration measured from the projection of first body, then $t = \frac{k}{g}$ sec, find k. (where g is gravitation acceleration)

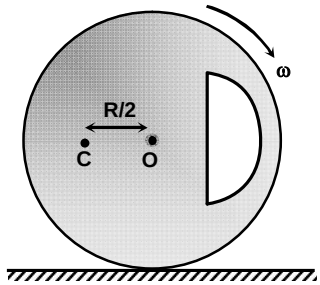
16. A car can pull a trailer of twice its mass up a certain slope at a maximum speed v = 1 m/s. Without the trailer the maximum speed of the car up the same slope is 2v. The resistance to the motion is proportional to mass and square of speed. If the car (without trailer) starts to move down the same slope, with its engine shut off, eventually it will acquire a constant speed v_t, then find v_t^2 (in m^2/s^2).

Space for Rough work

17. A dumbbell consists of two balls A and B each of mass m = 1 kg and connected by a spring. The whole system is placed on a smooth horizontal surface as shown in the figure. Initially the spring is at its natural length, the ball B is imparted a velocity $v_0 = \frac{8}{\sqrt{7}}$ m/s in the direction shown. The spring constant of the spring is adjusted so that the length of the spring at maximum elongation is twice that of the natural length of the spring. Find the maximum potential energy stored (in Joule) in the spring during the motion.

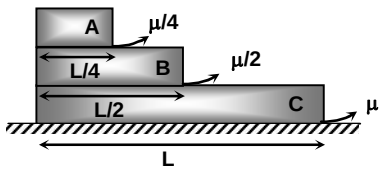


18. A disc shaped body (having a hole as shown in the figure) of mass m = 10 kg and radius R = $\frac{10}{9}$m is performing pure rolling motion on a rough horizontal surface. In the figure point O is geometrical center of the disc and at this instant the centre of mass C of the disc is at same horizontal level with O. The radius of gyration of the disc about an axis passing through C and perpendicular to the plane of the disc is $\frac{R}{2}$ and at the instant shown the angular velocity of the disc is $\omega = \sqrt{\frac{g}{R}}$ rad/sec in clockwise sense. g is gravitation acceleration = 10 m/s^2. Find angular acceleration α(in rad/s^2) of the disc at this instant.



19. A long plank begins to move at t = 0 and accelerates along a straight track with a speed given by $v = 2t^2$ for $0 \le t \le 2$ (where v is in m/s and t is in second). After 2 sec the plank continues to move at the constant speed acquired. A small block initially at rest on the plank begins to slip at t = 1 sec and stops sliding at t = 3 sec. If the coefficient of static friction and kinetic friction between the plank and the block is 0.s and 0.k (where s and k are digits) respectively, find s + k. (take g = 10m/s^2)

20. Three blocks A, B and C of mass m, $\frac{m}{2}$ and m of different densities and dimensions are placed over each other as shown in the figure. The coefficients of friction are shown. Blocks placed in a vertical line are made to move towards right with same velocity at the same instant. Find the time (in sec) taken by the upper block A to topple from the middle block B. Assume that blocks B and C don't stop sliding before A topples from B. (given L = 36 m, μ = 0.4 and g = 10 m/s^2)

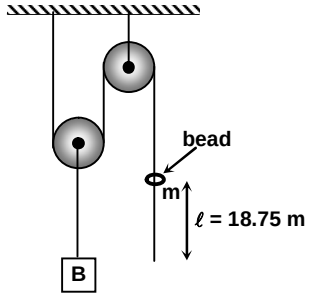


Space for Rough work

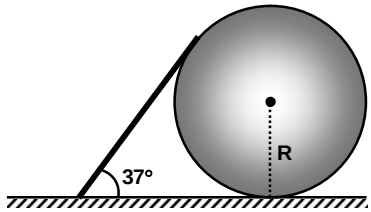
SECTION – D
(Numerical Based XXXXX.XX answer Type)

This section contains **3 questions**. Each question, when worked out will result in numerical answer Type with answer xxxxx.xx.

21. In the system shown in the figure, a bead of mass m can slide on the string. There is friction between the bead and the string. Block B has mass equal to twice that of the bead. The system is released from rest with length ℓ = 18.75 m of the string hanging below the bead. Assuming the pulley and string to be massless. Find the distance (in meter) moved by the block B before the bead slips out of the thread.



22. A stick of mass density λ = 8 kg/m rests on a disc of radius R = 20 cm as shown in the figure. The stick makes an angle θ = 37° with the horizontal and is tangent to the disc at its upper end. Friction exists at all points of contact and assume that it is large enough to keep the system at rest. Find the friction force (in Newton) between the ground and the disc. (take g = 10 m/s^2)



23. A bead moves along straight horizontal wire of length L, starting from the left end with a velocity v_0. Its retardation is proportional to the distance from the right end of the wire. Find the initial retardation (in m/s^2) (at left end of the wire) if the bead reaches the right end of the wire with a velocity $\frac{v_0}{2}$. (given v_0 = 5 m/s and L = 1 m)

Space for Rough work

Chemistry

PART – II

SECTION – A
(One Options Correct Type)

This section contains **3 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

24. 10 ml of $\frac{M}{10}$ NH_4OH is mixed with 4 ml of $\frac{M}{10}$ H_2SO_4 solution. The pH of the resulting solution is (pK_b NH_4OH = 4.76), (log2 = 0.3).
(A) 5.36 (B) 8.64
(C) 9.26 (D) 4.74

25. A catalyst lower the activation energy of a reaction from 200 KJ mol^{-1} to 100 KJ mol^{-1}. The temperature at which the rate of uncatalysed reaction will be same as the rate of catalysed reaction at 25^o C is (assuming Arrheneous constant A to be same in both the cases)
(A) 50^oC (B) 225^oC
(C) 323^oC (D) 596^oC

26. Which of the following reaction is incorrect?
(A) $2B_2O_3 + P_4O_{10} \longrightarrow 4BPO_4$ (B) $BCl_3 + 3H_2O \longrightarrow H_3BO_3 + 3HCl$
(C) $2BF_3 + 6NaH \xrightarrow{180^oC} 2B + 6NaF + 3H_2$ (D) $B_3N_3H_6 + 3HCl \longrightarrow B_3N_3H_9Cl_3$

(One or More than one correct type)

This section contains **FIVE** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

27. Which of the following statement is/are correct?
(A) In PCl_3F_2 the P – F bond length is greater than P – Cl bond length.
(B) N – N bond length in N_2H_4 is greater than N – N bond length in N_2F_4.
(C) In PCl_5 the P – Cl axial bond length is greater than P – Cl equatorial bond length.
(D) In SF_6 all S – F bonds have equal bond length.

28. The **incorrect** statement(s) among the following is/are?
(A) In aqueous solution Na has stronger reducing character than Li.
(B) LiH is thermally more stable than NaH.
(C) Li_2CO_3 is thermally more stable than Na_2CO_3.
(D) KO_2 is paramagnetic in nature.

Space for Rough work

29. Which of the following silicates is/are pyrosilicate?
(A) $Sc_2Si_2O_7$ (B) $MgSiO_3$
(C) $Ca_3Si_3O_9$ (D) Zn_2SiO_4

30. Which of the following statement(s) is/are correct?
(A) Borax on strong heating produces a glassy mass of sodium metaborate and boric anhydride.
(B) The formula of borax is $Na_2[B_4O_5(OH)_2].8H_2O$.
(C) Aqueous solution of borax is weakly acidic in nature.
(D) The number of B – O – B bonds in borax is 5.

31. Which of the following statement(s) is/are correct for the following equilibrium?
$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$
(A) Addition of $NH_4HS(s)$ at equilibrium increases the concentration of $NH_3(g)$.
(B) Addition of $NH_3(g)$ at equilibrium decreases the concentration of $H_2S(g)$.
(C) K_P is independent of the amount of $NH_4HS(s)$.
(D) Increase in temperature changes the K_P of this reaction.

Space for Rough work

(Paragraph Type)

This section contains **ONE** paragraph. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 32 to 33

Solid $AgNO_3$ is gradually added to a solution which is 0.01 M in Cl^- and 0.01 M in CO_3^{2-}.

$K_{sp}AgCl = 1.8 \times 10^{-10}$ and $K_{sp}Ag_2CO_3 = 4 \times 10^{-12}$.

32. The minimum concentration of Ag^+ required to start the precipitation of Ag_2CO_3 is
(A) 2×10^{-5} M (B) 4×10^{-10} M
(C) 4×10^{-5} M (D) 2×10^{-10} M

33. The concentration of Cl^- when Ag_2CO_3 starts precipitating is
(A) 1.8×10^{-9} M (B) 1.8×10^{-5} M
(C) 1.2×10^{-4} M (D) 9×10^{-6} M

Space for Rough work

SECTION – C
(Single digit integer type)

This section contains **TEN** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

34. In a hydrogen atom the de-Broglie wavelength of an electron is 1.67 nm. The value of principal quantum number of the electron is

35. For the redox reaction
$xCr(OH)_3 + yIO_3^- + zOH^- \longrightarrow xCrO_4^{2-} + yI^- + 5H_2O$
the sum of the stoichiometric coefficient (x + y + z) is equal to

36. The following equilibrium exists in a closed vessel in 1 L capacity
$A(g) + 3B(g) \rightleftharpoons 4C(g)$
initial concentration of A(g) is equal to that of B(g). The equilibrium concentration of A(g) and C(g) are equal. K_c for the reaction is

37. A vessel at 1000 K contains $CO_2(g)$ at 2 atm pressure. When graphite is added the following equilibrium is established
$CO_2(g) + C(s) \rightleftharpoons 2CO(g)$
the total pressure at equilibrium is 3 atm. The value of K_p is

38. The half-lie period for the decomposition of $AB_2(g)$ at 100 mm pressure is 10 min and at 200 mm pressure is 5 min, the order of reaction is

39. How many of the following compounds have polymeric structure?
Boron Nitride, Boric acid, Borazole, Borax, Beryllium hydride, Gypsum, Graphite.

40. Total number of solutions from the following which has pH < 7 at 25°C.
$\left(K_a CH_3COOH = 1.8 \times 10^{-5}, K_b NH_4OH = 1.8 \times 10^{-5}\right)$
(i) 10^{-8} M HCl
(ii) 0.01 M solution of NH_4Cl
(iii) 0.01 M solution of CH_3COONa
(iv) 0.01 M solutin of $B(OH)_3$
(v) 0.01 M solution of CH_3COONH_4

Space for Rough work

41. 200 ml of 1 M CH_3COOH ($K_a = 10^{-6}$) is mixed with 200 ml of 0.1 M HCOOH ($K_a = 10^{-5}$). The pH of the resulting mixture is

42. How many of the following molecules/ion are planar?
$CO_3^{2-}, NO_3^-, XeF_4, XeO_4, SF_4, I_3^-, COCl_2, SO_3, ClO_3^-, I_2Cl_6$ (solid)

43. How many of the following hydroxides are soluble in excess NaOH solution?
$Fe(OH)_3$, $Al(OH)_3$, $Zn(OH)_2$, $Ni(OH)_2$, $Mn(OH)_2$, $Sn(OH)_2$, $Cu(OH)_2$

SECTION – D
(Numerical Based XXXXX.XX answer Type)

This section contains **3 questions**. Each question, when worked out will result in numerical answer Type with answer xxxxx.xx.

44. The shortest wavelength of transition in Paschen series of He^+ ion in nanometer(nm) is $\left(\frac{1}{R_H} = 91.12\text{nm}\right)$

45. 4 mole of $S_2Cl_4(g)$ is introduced into a 10 L vessel. The following equilibrium was established
$S_2Cl_4(g) \rightleftharpoons 2SCl_2(g)$
at equilibrium 0.2 mol of S_2Cl_4 was present in the vessel. The value of equilibrium constant is.

46. A weak base MOH was titrated against a strong acid. The pH at $\frac{1}{4}$th equivalence point was 9.3. What will be the pH at $\frac{3}{4}$th equivalence point in the same titration? (log3 = 0.48)

Space for Rough work

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

This section contains **3 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

47. If $\int_{-20}^{-10}\left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx+\int_{\frac{1}{21}}^{\frac{1}{11}}\left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx+\int_{\frac{21}{20}}^{\frac{11}{10}}\left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx=\ell$ then $\ell+\frac{420}{7939}$ is equal to

(A) $\frac{110}{939}$ (B) $\frac{110}{969}$

(C) $\frac{110}{739}$ (D) $\frac{120}{759}$

48. Solution of the differential equation $x^2dy - 2xydx = x^3y^3dx + x^4y^2dy$ is

(A) $y = kx^2e^{\frac{x^2y^2}{2}}$ (B) $y = ky^2e^{\frac{x^2y^2}{2}}$

(C) $y=\frac{k}{x^2}e^{\frac{3x^2y^2}{2}}$ (D) $\ln\left(\frac{y}{x^2}\right)=\frac{(xy)^2}{4}+e^{c^2}$

49. If $f(x) = k^3x + k^3 - 2$ cuts the curve $g(x) = \frac{1}{2}\ln x^2$ at exactly one point then 'k' may lie in the interval

(A) $\left(\frac{1}{\sqrt{e}}, e\right)$ (B) $\left(\frac{1}{e}, \frac{1}{\sqrt{e}}\right)$

(C) $\left(\frac{1}{e^2}, \frac{1}{e}\right)$ (D) none of these

Space for rough work

(One or More than one correct type)

This section contains **FIVE** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

50. Let y = f(x) be the solution of the differential equation $y' = \frac{3y^2 + x}{4y^2 + 5}$ where $y\left(\frac{15}{4}\right) = 0$ then which of the following is/are correct?

(A) $y' \geq \frac{3}{4} \forall\ x \geq \frac{15}{4}$ (B) $\int_{15/4}^{27/4} f(x)\,dx \geq \frac{27}{8}$

(C) $\int_{15/4}^{27/4} f(x)\,dx < \frac{27}{8}$ (D) $f'(x) \geq 0\ \forall\ x \geq 0$

51. The solution of the differential equation $(x^2 + 4y^2 - 5)xdx = (4x^2 - 3y^2 - 1)ydy$ is

(A) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\left(\frac{y^2-1}{x^2-1}\right)\right) - \frac{1}{4}\ln\left(6\left(\frac{y^2-1}{x^2-1}\right)^2 + 2\right) - \ln\sqrt{|x^2-1|} + c = 0$

(B) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\left(\frac{y^2-1}{x^2-1}\right)\right) - \frac{1}{4}\ln\left(6\left(\frac{y^2-1}{x^2-1}\right)^2 + 2\right) - \ln\sqrt{|x^2-1|} + \tan c = 0$

(C) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\left(\frac{x^2-1}{y^2-1}\right)\right) - \frac{1}{4}\ln\left(6\left(\frac{y^2-1}{x^2-1}\right)^2 + 2\right) - \ln\sqrt{|x^2-1|} + e^c = 0$

(D) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\left(\frac{x^2-1}{y^2-1}\right)\right) - \frac{1}{4}\ln\left(6\left(\frac{x^2-1}{y^2-2}\right)^2 + 2\right) - \ln\sqrt{|x^2-1|} + e^c = 0$

52. Let functions are defined from set A to set B where B = $\{\alpha, \beta\}$ and α & β are the roots of the equation $t^2 - \sqrt{2}t - \pi = 0$, then the number of functions which are

(A) discontinuous only at each even integers if A = [0, 11] is 682
(B) discontinuous only at each odd integer if A = [0, 11] is 243
(C) discontinuous only at prime numbers if A = [0, 11] is 81
(D) discontinuous only at $x = 5k(k \in I^+)$ if A = [0, 11] is 27

53. If f(x) is a twice differentiable function and given that f(1) = 2, f(2) = 5 and f(3) = 10 then

(A) $f''(x) = 2\ \forall\ x \in (1, 3)$ (B) $f''(x) = f'(x) = 2$ for some $x \in (2, 3)$
(C) $f''(x) = 3\ \forall\ x \in (2, 3)$ (D) $f''(x) = 2$ for some $x \in (1, 3)$

Space for rough work

54. Let $f(x)=\begin{cases}0 & ; \text{ x is irrational}\\ \dfrac{2}{2q^3-q^2+q+\sin^2 q+5} & ; \text{ if } x=\dfrac{p}{q}\text{ (rational) }(\text{where HCF }(p, q)=1;\ p, q>0)\end{cases}$ and f(x) is defined $\forall$ x > 0 then which of the following is/are incorrect?

(A) f(x) is continuous at each irrational in (0, ∞)
(B) f(x) is continuous at each rational in (0, ∞)
(C) f(x) is discontinuous at each rational in (0, ∞)
(D) f(x) is discontinuous for all x in (0, ∞)

(Paragraph Type)

This section contains **ONE** paragraph. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 55 to 56

Read the following write up carefully and answer the following questions:

Let y = f(x) be the solution of the differential equation $\frac{dy}{dx}+\frac{k}{7}x\tan x=1+x\tan x-\sin x$, where f(0) = 1 and let k be the minimum value of g(x) where $g(x)=\max\left|\frac{\sqrt{193}-1}{2}\cos y+\cos\left(y+\frac{\pi}{3}\right)-x\right|$ where $y \in R$, then

55. Area bounded by y = f(x) and its inverse between $x=\frac{\pi}{2}$ and $x=\frac{7\pi}{2}$ is

(A) 12 (B) 6
(C) 9 (D) 8

56. Number of solution of the equation $f(x) = 2^x - x^2 + x + \cos x$ is equal to

(A) 3 (B) 1
(C) 2 (D) 4

Space for rough work

SECTION – C
(Single digit integer type)

This section contains **TEN** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

57. Let $f(x)=\begin{cases}|x-1|+|x-2| & ; \ x\geq 1\\ x & ; \ x<1\end{cases}$ and $g(x)=\begin{cases}\max\{f(t): x-1\leq t\leq x\} & : \ 0\leq x\leq 2\\ -x+3 & : \ 2<x\leq 3\end{cases}$ then number of points in [0, 3] where g(x) is not differentiable is/are _____

58. Let Q(x) be a function defined for $x \in [e^3, e^6]$ be a real valued differentiable function such that $Q(e^3) = 1$ and $Q'(x) = \dfrac{2}{x+\ln\left(\ln x+\dfrac{3}{\ln x}+e-4\right)}$ then maximum value of Q can't exceed a number ℓ ($\ell \in N$), then minimum value of ℓ is _____

59. If $\lim\limits_{x\to 1^-}\prod\limits_{n=0}^{\infty}\left(\dfrac{1+x^{n+1}}{1+x^n}\right)^{x^n} = \ell$ then $\left[\dfrac{1}{\ell}\right]$ is equal to _____

60. If the least area bounded by the curves $y = x^2 - 4$ and $y = \lambda x + 12$ is equal to $\dfrac{\alpha}{\beta}$, then $\left[\dfrac{\alpha}{20\beta}\right]$ is equal to _____ (where [.] denotes the greatest integer function)

61. The range of real constant 't' for which $(1 - \tan^2 t)\sin\theta^2 + \tan^2 t\cdot\tan\theta^2 \geq \theta^2$; always holds $\forall\ \theta\in\left(0,\dfrac{\pi}{2}\right)$ is $[\alpha, \beta)$ then $\dfrac{\beta}{\alpha}$ is equal to _____

62. The complete set of non-zero values of 'k' such that the equation $|x^2 - 7x + 6| = kx$ is satisfies by at least one and at most three real value(s) of x is $(-\infty, \ell_1] \cup [\ell_2, \infty)$ then $\dfrac{|\ell_1-\ell_2|}{2}$ is equal to _____

63. Let f be the real valued differentiable function on R such that $e^{-x}f(x)=\dfrac{3}{e^2}+4e^{-x}\int\limits_2^x\sqrt{2t^2+6t+5}\,dt$ $\forall\ x \in R$ and let $g(x) = f^{-1}(x)$ then $[g'(3)] + [|g''(3)|]$ is equal to _____ (where [.] denote the greatest integer function)

Space for rough work

64. Let $f(x) = [x] + \{x\}^3$ then the area of the figure bounded by $y = f^{-1}(x)$, $y = 0$ between the ordinates $x = 2$ and $x = \frac{9}{2}$ is α, then $\alpha - \frac{3}{2^{10/3}} + \frac{1}{2}$ is equal to _____ (where [.] denotes the greatest integer function)

65. Total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^8+1}{t^8+t^2+1}dt = 3x - 2$ is _____

66. Let

$f(x) = ax^{17} + b \sin x \cdot \sin 2x \cdot \sin 3x + cx^2 \operatorname{sgn}(\sin x) + d \log\left(x + \sqrt{1+x^2}\right) + x\left(|x+1| - |x-1|\right)\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$

be defined on the set of real numbers, $(a > 0, b, c, d \in R)$ if $f(-7) = 7$, $f(-5) = -5$, $f(-2) = 3$, then the minimum number of zeroes of the equation $f(x) = 0$ is equal to _____

SECTION – D
(Numerical Based XXXXX.XX answer Type)

This section contains **3 questions**. Each question, when worked out will result in numerical answer Type with answer xxxxx.xx.

67. If $I_1 = \int_0^1 \left(1-\left(1-x^3\right)^{\sqrt{2}}\right)^{\sqrt{3}} x^2 dx$ and $I_2 = \int_0^1 \left(1-\left(1-x^3\right)^{\sqrt{2}}\right)^{\sqrt{3}+1} \cdot x^2 dx$, then $\frac{\frac{I_1}{I_2} - \frac{\sqrt{3}-1}{2\sqrt{2}} + 0.2}{10}$ is equal to _____

68. If $f(x) = 2x^3 - 3x^2 + 1$ then number of distinct real solution(s) of the equation $f(f(x)) = 0$ is/(are) k then $\frac{7k}{10^2}$ is equal to _____

69. Number of points of inflexion on the curve $f(x) = (x-1)^7(x+2)^8$ is equal to ℓ then $\frac{5\ell}{10^2}$ is equal to _____

Space for rough work