FIITJEE

JEE(Advanced)-2018

ALL INDIA TEST SERIES

ANSWERS, HINTS & SOLUTIONS

PART TEST – I

PAPER-1

ANSWERS KEY

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	**A, B**	19.	**A, C**	37.	**A, B, C, D**
2.	**B, C**	20.	**A, C**	38.	**A, B, C, D**
3.	**A, C**	21.	**A, B, C, D**	39.	**B, C**
4.	**B, C**	22.	**A, B, C, D**	40.	**A, C**
5.	**A, C, D**	23.	**B, C**	41.	**A, C**
6.	**B, C**	24.	**A, B**	42.	**A, B, C**
7.	**A, C**	25.	**A, C, D**	43.	**A, B, D**
8.	**C**	26.	**B**	44.	**C**
9.	**B**	27.	**A**	45.	**D**
10.	**D**	28.	**D**	46.	**A**
11.	**B**	29.	**B**	47.	**B**
12.	**A**	30.	**A**	48.	**C**
13.	**C**	31.	**D**	49.	**C**
14.	**4**	32.	**5**	50.	**4**
15.	**9**	33.	**4**	51.	**0**
16.	**4**	34.	**5**	52.	**9**
17.	**8**	35.	**4**	53.	**1**
18.	**8**	36.	**2**	54.	**5**

Physics

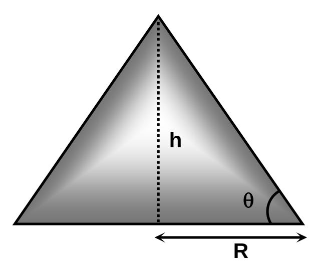
PART – I

SECTION – A

1. $20 - T_1 = 2a$ (for 2 kg block) ….(i)
$T_2 - 10 = 1a$ (for 1 kg block) ….(ii)
$T_1\frac{1}{2} - T_2\frac{1}{2} = (2)\left(\frac{1}{2}\right)^2\frac{1}{2}(2)\frac{a}{1/2}$ (for the shaft + disc system)
So, $T_1 - T_2 = 2a$ ….(iii)
After solving these three equation
$a = 2\ m/s^2$

2. $v_{CM} = \omega\sqrt{R^2 - \frac{3R^2}{4}} = \omega\frac{R}{2}$ (from pure rolling)
$mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\omega^2$ (from conservation of energy)
$\therefore\ v_{CM} = \sqrt{\frac{10gh}{13}}$

3. From free body diagram of sand particle
$\tan\theta = \mu_S = h/R$
work required = $\left(\frac{1}{3}\pi R^2 h\rho g\right)\frac{h}{4}$.
The system is in unstable equilibrium.



4. For small angle of inclinations, ball rolls without slipping. So there is no work done by friction. At some critical angle θ_C, the ball starts slipping. For $\theta > \theta_C$, energy would be lost due to work done by kinetic friction. But again for ($\theta = \pi/2$), friction would be zero.

5. As going up, speed of the particle is decreasing and hence the time taken in crossing the windows will be $t_1 < t_2 < t_3$ (If $W_1 = W_2 = W_3$)
Simultaneously $\Delta V \propto t$
$\therefore\ \Delta V_1 < \Delta V_2 < \Delta V_3$
For unequal windows $t_1 = t_2 = t_3$ may be if $W_3 < W_2 < W_1$

6. $F = -\frac{dU}{dr} = \frac{A}{r^3} - \frac{B}{3r^2}$
For equilibrium F = 0, r = $\frac{3A}{B}$
$\frac{B^2}{6A} = \frac{A}{2r_0^2} - \frac{B}{3r_0}$ (for $r_0 = \frac{r}{3}$)

7. $\Delta\vec{P} = 2(\hat{i} - \hat{j}) - 2(-2\hat{i} - 2\hat{j})$
$= 6\hat{i} + 2\hat{j}$ kg-m/s

So, $\Delta\vec{P}$ makes angle $\tan^{-1}\left(\frac{1}{3}\right)$ with positive x-axis which is also the direction of normal force exerted on the ball. So the plane surface makes an angle $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$ with positive x-axis.

8. Center of mass of a triangular uniform plate from its base is $\frac{h}{3}=1\text{m}$

9. $I_{axis} = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 = \frac{7}{5}\times\frac{5}{7}\times 9 = 9$ kg-m^2

10. $\frac{U}{g} = Mh_{CM} = (2)\left(\frac{3}{3}\right) = 2\,\text{m}$

11. For elevator acceleration 1.5 m/s^2, tension in cord 2 and cord 3 is zero
So, $T_1 - 150 = (15)(1.5)$
$\Rightarrow$ Tension in cord 1, $T_1 = 172.5$ N

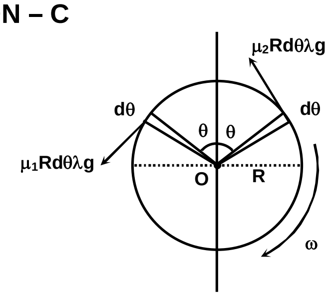
12. For elevator acceleration 1 m/s^2, tension in cord 2 and cord 3 decreases.
So, $165 + \Delta T - 2\times(7.5-\Delta T) - 150 = (15)(1)$
$\Rightarrow \Delta T = 5$ N
$\Rightarrow$ Tension in cord 2, $T_2 = 2.5$ N

13. For elevator acceleration 2 m/s^2, tension in cord 2 and cord 3 is zero.
So, $T_1 - 150 = (15)(2)$
$\Rightarrow$ Tension in cord 1, $T_1 = 180$ N
So, displacement of block = 15 cm downward.

SECTION – C

14. $$2\left[\int_0^{\pi/2}(\mu_1-\mu_2)Rg\lambda\cos\theta\,d\theta\right] = 2(\mu_1-\mu_2)R\lambda g$$

$$\therefore\ a = \frac{2(\mu_1-\mu_2)R\lambda g}{2\pi R\lambda} = 4$$



15. At v_{max}

$$Av_{max}^2 = F = \frac{km}{v_{max}}$$ (where k is a proportionalty constant)

$$m^{2/3}v_{max}^2 \propto \frac{m}{v_{max}}$$

$$\therefore\ v_{max} \propto m^{1/9}$$

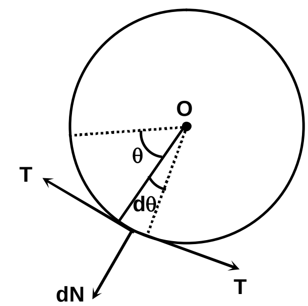
16. $1.25 = \frac{1}{2}(10)t^2$
$\therefore$ t = 0.5 sec
Distance of the cylinder from the truck = $8\times 0.5 = 4$ m

17. $2T\sin\frac{d\theta}{2} - dN = \lambda R d\theta \frac{v^2}{R}$ (λ is linear mass density of belt)

$\therefore\ Td\theta - dN = \lambda v^2 d\theta$

$\therefore$ so total normal = $\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} dN\cos\theta = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} T\cos\theta d\theta - \lambda v^2 \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta d\theta$

$\therefore$ N = 8 newton



18. $F = 2m\eta A\left[(v_0+v)^2-(v_0-v)^2\right]$

$= 2(10^{-26})(10^{25})(1)\left[(5+2)^2-(5-2)^2\right]$

$= 0.2 \times 40 = 8$ N

Chemistry

PART – II

SECTION – A

19. $V \propto n$

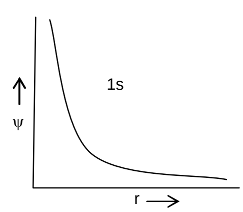
$P_{PCl_3} = P_{Cl_2} = 2\times\frac{40}{100} = 0.8$ atm

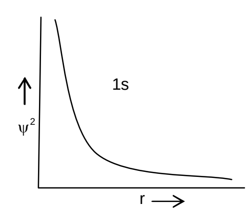
$K_P = \frac{0.8\times0.8}{0.40} = 1.6$ atm

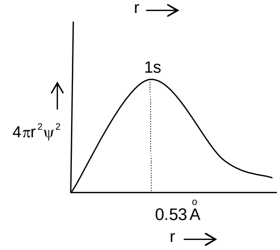
$K_C = \frac{K_P}{(RT)^{\Delta n}} = \frac{1.6}{0.0821\times520} = 0.037$

Increase in pressure increases the number of moles PCl_5(g).

20.







21. All the reactions are disproportionation reactions.

22. $CaC_2 + 2H_2O \longrightarrow Ca(OH)_2 + C_2H_2$

$CaC_2 + N_2 \longrightarrow Ca(NCN) + C$

$Be_2C + 4H_2O \longrightarrow 2Be(OH)_2 + CH_4$

Cyanamide ion $[N = C = N]^{2-}$ is isoelectronic with CO_2.

23.

$$NO(g) + NO_2(g) \rightleftharpoons N_2O_3(g)$$

$$\begin{array}{lll} 3P & 5P & 0 \\ 3P-x & 5P-x-2y & x \end{array}$$

$$2NO_2(g) \rightleftharpoons N_2O_4(g)$$

$$\begin{array}{ll} 5P-x-2y & y \end{array}$$

$$K_{P_2} = \frac{P_{N_2O_4}}{\left(P_{NO_2}\right)^2}, \quad 8 = \frac{P_{N_2O_4}}{(0.5)^2}$$

$$P_{N_2O_4} = 2 \text{ atm}$$

$$\therefore y = 2 \text{ atm}$$

$$P_{total} = P_{NO} + P_{NO_2} + P_{N_2O_3} + P_{N_2O_4}$$

$$5.5 = 3P - x + 0.5 + x + 2$$

3P = 3, P = 1

5P – x – 2y = 0.5

5 × 1 – x – 2 × 2 = 0.5, x = 0.5 atm.

$$K_{P_1} = \frac{P_{N_2O_3}}{(P_{NO})(P_{NO_2})}$$

$$= \frac{0.5}{2.5 \times 0.5} = 0.4 \text{ atm}$$

$$P_{NO} = 2.5 \text{ atm } P_{NO_2} = 0.5 \text{ atm } P_{N_2O_3} = 0.5 \text{ atm}, P_{N_2O_4} = 2 \text{atm}$$

24. B – F bond in BF_3 is smaller due to $P\pi - P\pi$ back bonding.

$$B_3N_3H_6 + 9H_2O \longrightarrow 3NH_3 + 3H_3BO_3 + 3H_2$$

In $T\ell I_3$ oxidation state of $T\ell$ is +1.

25.

$$\underset{\substack{60\times0.1=6 \\ 2}}{NH_4OH} + \underset{\substack{40\times0.1=4 \\ 0}}{HCl} \longrightarrow \underset{\substack{0 \\ 4}}{NH_4Cl} + H_2O$$

The resulting solution is a basic buffer.

$$pOH = pK_b + \log\frac{\left[NH_4^+\right]}{[NH_4OH]}$$

$$pOH = 4.74 + \log\frac{4}{2} = 5.04$$

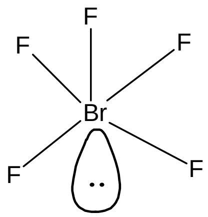
$$pH = 14 - 5.04 = 8.96$$

26. Decomposition of HI on surface of gold is a zero order reaction.

27. For a 1st order reaction $k = \frac{2.303}{t}\log\frac{a}{a-x}$.

28. For 2nd order reaction $k = \frac{1}{t}\left(\frac{1}{a-x} - \frac{1}{a}\right)$.

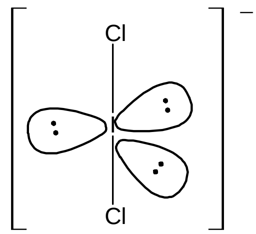
Solution for the Q. No. 29 to 31.



Hybridisation – sp^3d^2

Shape – square pyramidal
Lone pair in central atom = 1

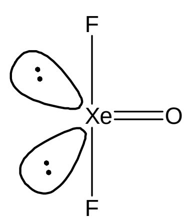
$[ICl_2]^-$



Hybridisation – sp^3d

Shape – Linear
Lone pair in the central atom = 3

$XeOF_2$



Hybridisation – sp^3d

Shape – T-shaped
Lone pair in the central atom = 2

SECTION – C

32. $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$

1 mol of $LiNO_3$ will produce NO_2 = 1

$2Mg(NO_3)_2 \xrightarrow{\Delta} 2MgO + 4NO_2 + O_2$

1 mol of $Mg(NO_3)_2$ will produce NO_2 = 2

$2Ca(NO_3)_2 \xrightarrow{\Delta} 2CaO + 4NO_2 + O_2$

1 mol of $Ca(NO_3)_2$ will produce NO_2 = 2

$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

Total number of moles of NO_2 produced = 5

33. $V = 2.188 \times 10^6 \times \frac{Z}{n} m/sec.$

$\therefore n = \frac{2.188 \times 10^6 \times 1}{5.47 \times 10^5} = 4$

$\therefore$ According to de-Broglie, total number of waves made by an electron in one complete revolution = n = 4

34. The following species are paramagnetic.
$N_2^+, O_2^-, B_2, H_2^-, He_2^+$

35. $\underset{\substack{6\times0.1=0.6\\0.6-0.x}}{CH_3COOH} + \underset{x\times0.1}{NaOH} \longrightarrow \underset{\substack{0\\0.x}}{CH_3COONa} + H_2O$

The resulting solution is a buffer solution.

$$pH = pK_a + \log\frac{[CH_3COO^-]}{[CH_3COOH]}$$

$$pH = pK_a + \log\frac{0.x}{0.6-0.x}$$

$$5.04 = 4.74 + \log\frac{0.x}{0.6-0.x}$$

$$0.3 = \log\frac{0.x}{0.6-0.x}$$

$$\log 2 = \log\frac{0.x}{0.6-0.x}$$

$$\frac{0.x}{0.6-0.x} = 2$$

$x = 4$

36. $\underset{10(1-\alpha)}{A(g)} \rightleftharpoons \underset{10\alpha}{B(g)} + \underset{10\alpha}{C(g)}$

$$D = \frac{100}{2} = 50, \quad d = \frac{83}{2} = 41.5$$

$$\therefore \alpha = \frac{D-d}{d(n-1)}$$

$$= \frac{50-41.5}{41.5(2-1)} = 0.2$$

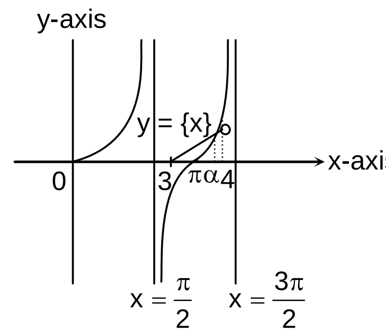
$\therefore$ Number of moles of 'C' at equilibrium $= 10\alpha = 10\times0.2 = 2$

Mathematics

PART – III

SECTION – A

37. Shown in the figure



38. (A) $$\frac{x-\frac{1}{x}}{4\left(x^2+\frac{1}{x^2}-1\right)} = \frac{x-\frac{1}{x}}{4\left((x-1/x)^2+1\right)}$$

So, $$-\frac{1}{8} \le \frac{x-\frac{1}{x}}{4\left(\left(x-\frac{1}{x}\right)^2+1\right)} \le \frac{1}{8}$$

$$\therefore \left[\frac{x\left(x^2-1\right)}{4\left(x^4-x^2+1\right)}+\frac{1}{8}\right]=0$$

So: $f(x)=2x+[x]+\frac{1}{2}\sin 2x$ injective but not onto

(B) $f'(x) = -x^3 \cos x + x^2 (3 \sin x + 2 \cos x) + 4x(\sin x + 2)$
Many one onto

39. $T_n=\frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right)$ where $f(x) = x^4 + x^3 + x^2 + 2$, f(x) is an increasing function for $\forall$ x > 0.

$$T_n=\frac{1}{n}\left[f(0)+f\left(\frac{1}{n}\right).....+f\left(\frac{n-1}{n}\right)\right]$$

$$T_n<\int_0^1\left(x^4+x^3+x^2+2\right)dx=\frac{167}{60}$$

$$S_n=\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)>\frac{1}{n}\left[f\left(\frac{1}{n}\right)+.....+f\left(\frac{r}{n}\right)\right]=\frac{167}{60}$$

40. (A) Let $\int_0^{n\pi}\left|\frac{\sin t+\cos t}{t}\right|dt=A_n$

$$A_n>\frac{1}{\pi}\left[\int_0^{\pi}\left|\sin t+\cos t\right|dt+\int_{\pi}^{2\pi}\frac{\left|\sin t+\cos t\right|dt}{2}+.....\right]$$

$$>\frac{1}{\pi}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right)\left(\int_0^{\pi}\left|\sin t+\cos t\right|dt\right)$$

$$>\frac{2\sqrt{2}}{\pi}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right)$$

(C) $\frac{3\sqrt{\sin x}+8-3\sqrt{\sin x}}{2}\ge\sqrt{3\sqrt{\sin x}\left(8-3\sqrt{\sin x}\right)}$

(D) $1\le \tan x\le\sqrt{3}$ and $\frac{3}{\pi}\le\frac{1}{x}\le\frac{4}{\pi}$

So; $\frac{3}{\pi}\le\frac{\tan x}{x}\le\frac{4\sqrt{3}}{\pi}$

41. $\frac{1}{2}\operatorname{cosec}^2\left(x^2+y^2\right)d\left(x^2+y^2\right)=2\left(\frac{y}{x}\right)^3 d\left(\frac{y}{x}\right)$

$$\Rightarrow -\frac{1}{2}\cot\left(x^2+y^2\right)-\frac{2(y/x)^4}{4}+c=0$$

42. $I=\int_{1/8}^{7/8} f(f(x))dx$ ..... (1)

$I=\int_{1/8}^{7/8} f(f(1-x))dx=\int_{1/8}^{7/8} f(1-f(x))dx$ ..... (2)

So, $2I=\frac{3}{4}$

43. $\int e^{x\sec^2 x-\tan x}\left(x\tan x-\frac{\sin 2x}{2}\right)dx = \int e^{x\sec^2 x-\tan x}\left(\frac{\cos^2 x}{2}\left(x\sec^2 x-\tan x\right)' + \left(\frac{\cos^2 x}{2}\right)'\right)$

$= e^{x\sec^2 x-\tan x}\cdot\frac{\cos^2 x}{2}+c$

44.-46. (I) $f(x) = ||x-6|-|x-8|| - |x^2-4| + 3x - |x-7|^3$ is continuous $\forall\ x \in R$ and not differentiable at $x = -2, 2, 6, 7$ & 8

(II) $f(x) = (x^2-9)|x^2+11x+24| + \sin|x-7| + \cos|x-4| + (x-1)^{3/5}\sin(x-1)$ is continuous $\forall\ x \in R$ and not differentiable at $x = -8$ & 7

(III) $f(x) = \begin{cases} (x+1)^{3/5} - \frac{3\pi}{2} & : \ x < -1 \\ \left(x-\frac{1}{2}\right)\cos^{-1}\left(4x^3-3x\right) & : \ -1 \le x \le 1 \\ (x-1)^{5/3} & : \ 1 < x < 2 \end{cases}$ is discontinuous at $x = -1$ & 1 not differentiable at $x = -1, -\frac{1}{2}$ & 1

(IV) $f(x) = \{\sin x\}\{\cos x\} + (\sin^3\pi\{x\})([x]),\ x \in [-1, 2\pi]$

Let $g(x) = \underbrace{\left(\sin\pi\{x\}\right)\left([x]\right)}_{\text{cont. at } x = I}\left(\sin^2\pi\{x\}\right)$

$g'(I^+) = g'(I^-)$ so differentiable at $x = I$ and for $\{\sin x\}\{\cos x\}$

Doubtful points for non differentiability are $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\because$ $\{\sin x\}\cdot\{\cos x\}$ is discontinuous at $x = 0, \frac{\pi}{2}, 2\pi$

So not differentiable at $x = 2n\pi, 2n\pi + \frac{\pi}{2}$

47.-49. (I) $\frac{(3-1)^3}{4} \le |x-1|^3 + |x-3|^3 < \infty$

(II) $u = \sin\left(\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right) + \cos\left(\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$

$\because\ 0 \le \frac{\sqrt{4-x^2}}{1-x} < \infty$

$-\infty < \ln\left(\frac{\sqrt{4-x^2}}{1-x}\right) < \infty$

So; $-\sqrt{2} \le u \le \sqrt{2}$

(III) Let $|x+5| = t$; $g(t) \in \left(0, \frac{1}{2}\right] - \left\{\frac{1}{4}\right\}$

$g(t) = \frac{(t-2)}{(t-2)(t+2)}$

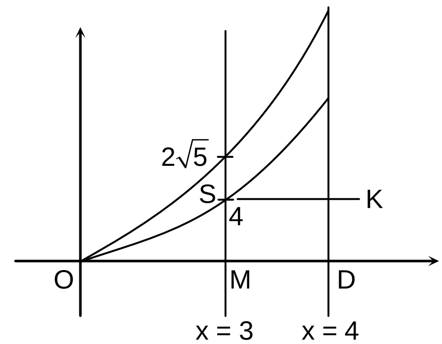
(IV) $\theta \le \cos^{-1} x \le \pi$

$-\infty < \ln(\cos^{-1} x) \le \ln\pi < 2$

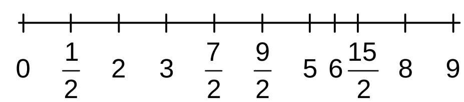
SECTION – C

50. Let cot A = a then $a^3 + a^2|a + x| + |a^2x + 1| = 1$
$|a^3 + a^2x| + |a^2x + 1| = (a^2x + 1) - (a^2x + a^3)$
$|\alpha| + |\beta| = \alpha - \beta$
So, $\alpha \geq 0$ and $\beta \leq 0$
Now take cases: $a \leq -1$ and $-1 < a \leq 0$ & $0 < a \leq 1$
Finally we get, $a \in (-\infty, -5] \cup \left[-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{6}}\right]$

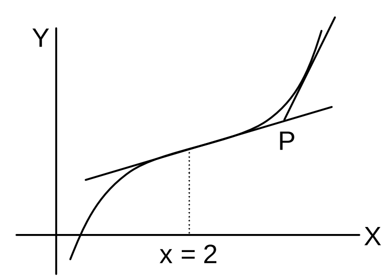
51. $\because\ 0 \leq f(x) \leq 2$
and $g(3) \in \left[-2\sqrt{5}, -4\right] \cup \left[4, 2\sqrt{5}\right]$
Also $\int_0^4 g(x)dx = \int_0^4 f'(x)dx = \left(f(4) - f(0) \in (-2, 2)\right)$
Case-I: Let $g(x) > 0$ and $g''(x) > 0$
Clearly $\int_0^4 g(x)dx \geq \text{Area}(MDKS) = 4$
Which is a contradiction
Thus, there is no such C
Similarly **case-II**: Let $g(x) < 0$ and $g''(x) < 0$



52. $f\left(\frac{x+13}{2}\right) = f\left(\frac{3-x}{2}\right)$
$f(x) = f(8 - x)$
$f'(x) = -f'(8 - x)$
$f'\left(\frac{1}{2}\right) = -f'\left(\frac{15}{2}\right) = 0$
$f'(2) = -f'(6) = 0$
$f'(3) = -f'(5) = 0$
$f'\left(\frac{9}{2}\right) = -f'\left(\frac{7}{2}\right) = 0$
$f'(0) = -f'(8)$; $h(x) = \frac{d}{dx}\left(f'(x)f''(x)\right)$
Clearly: h(x) has minimum 18 zeroes



53. $y'' = 0 \Rightarrow x = 2$
Let P be the point of inflection
So; $P \equiv (2, 2b - a - 16)$
Equation of tangent line passing through inflection
Point: $y = (b - 12)x - a + 8$ .....(1)
Let $Q \equiv (2 + h, 3h - 1)$
Locus of Q: $3x - y = 7$ ..... (2)
From equation (1) and (2), we get !
So; a = 15 and b = 15



54. $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\frac{\left(\lambda k^4 + 2k^3 + k^2 + k + 1\right)}{n^5}}{\frac{\left(3n^5 + n^2 + n + 5k\right)}{n^5}} = \int_0^1 \frac{\lambda x^4 dx}{3} = \frac{\lambda}{3}\left(\frac{x^5}{5}\right)_0^1 = \frac{\lambda}{15}$$