FIITJEE

JEE (Advanced)-2018

PART TEST – I

Paper 1

Time Allotted: 3 Hours **Maximum Marks: 183**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

ALL INDIA TEST SERIES

INSTRUCTIONS

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
4. Each part is further divided into Two sections: **Section-A & Section-C**
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with black pen for each character of your Enrolment No. and write your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

1. ***Section-A (01– 07, 19 – 25, 37 - 43)*** *contains 21 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer*
   Partial Marks **+1** for each correct option provided no incorrect options is selected.

   Section-A (08 – 13, 26 – 31, 44 - 49) *contains 18 questions. Each of 2 Tables with 3 Columns and 4 Rows has three questions. Column 1 will be with 4 rows designated (I), (II), (III) and (IV). Column 2 will be with 4 rows designated (i), (ii), (iii) and (iv). Column 3 will be with 4 rows designated (P), (Q), (R) and (S).*
   Each question has ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

2. ***Section-C (14 – 18, 32 – 36, 50 - 54)*** *contains 15 Numerical based questions with answer as numerical value from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

Name of the Candidate	
Enrolment No.	

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's constant	h	=	6.625×10^{-34} J·s
		=	6.625×10^{-27} erg·s
1 Faraday		=	96500 coulomb
1 calorie		=	4.2 joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

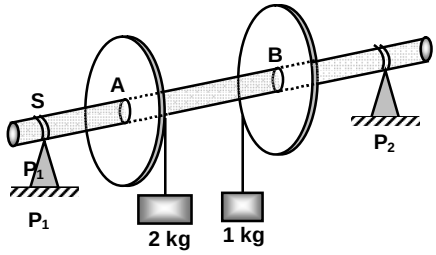
Physics

PART – I

SECTION – A
(One or More than one correct type)

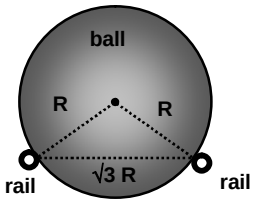
This section contains **7** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. Two discs A and B each of mass 2 kg and radius 0.5 m are rigidly fixed to the ends of a thin shaft S, which is supported in horizontal position with the help of two smooth bearings at points P_1 and P_2. Two blocks of masses 2 kg and 1 kg are connected to the light cords wrapped around the discs as shown in the figure. There is no slipping between the cord and the discs. If the system is released from rest from the position shown, then mark the correct statement(s). (Take g = 10 m/s^2)



(A) Power developed by tension in cord connected to 2 kg block is, negative.
(B) Acceleration of each block is 2 m/s^2.
(C) Total work done by tension on the discs is zero during the motion.
(D) Acceleration of each block is 1 m/s^2.

2. A uniform ball of radius R rolls without slipping between the rails such that the horizontal distance is $\sqrt{3}$ R between the two contact points of the rails of the ball. Figure (a) shows the front view of the ball and figure (b) shows the side view of the ball. v_{CM} is the velocity of centre of mass of the ball and ω is the angular velocity of the ball after rolling down a distance 2h along the incline then



(velocity into the plane)

figure (a)

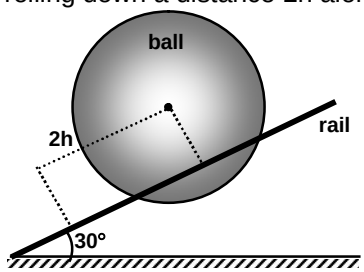


figure (b)

(A) $v_{CM} = \omega R$

(B) $v_{CM} = \omega \frac{R}{2}$

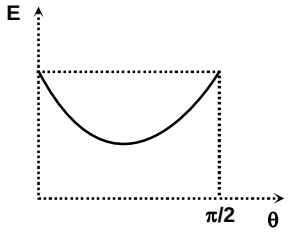
(C) $v_{CM} = \sqrt{\frac{10gh}{13}}$

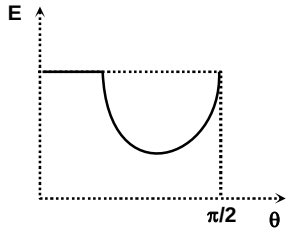
(D) $v_{CM} = \sqrt{\frac{10gh}{7}}$

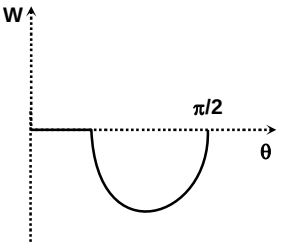
Space for Rough work

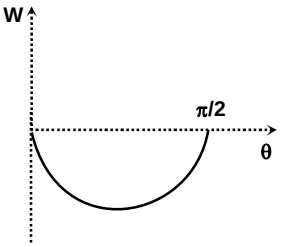
3. A worker wishes to pile up sand on to a circular area of radius R. No sand is to spill on to the surrounding area. If μ_s is the coefficient of static friction between sand particles, then choose the correct statement(s).
(A) The greatest height of sand pile that can be created is $\mu_s R$.
(B) The greatest height of sand pile that can be created is R/μ_s.
(C) Minimum work required to create greatest height sand pile is $\frac{\pi}{12}\mu_s^2 R^4 \rho g$. Where ρ is the density of sand volume.
(D) Sand pile will be in stable equilibrium.

4. A solid spherical ball is released from rest on an incline of inclination angle θ (which can be varied) but through a fixed vertical height h. The coefficient of static and kinetic friction are both equal to μ. If E represents the total kinetic energy of the ball at the bottom of the incline as a function of the angle of inclination θ. W represents the work done by friction for the whole time of motion as a function of the angle of inclination θ. Choose the correct graph(s).

(A) 

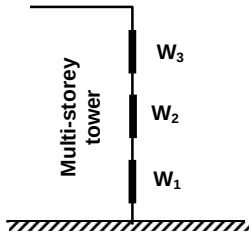
(B) 

(C) 

(D) 

Space for Rough work

5. W_1, W_2 and W_3 are the different sizes of windows 1, 2 and 3 respectively placed in a vertical plane. A particle is thrown up in the vertical plane. Let t_1, t_2 and t_3 are the time taken to cross the window W_1, W_2 and W_3 respectively and ΔV_1, ΔV_2 and ΔV_3 are the change in speed after respective window cross.



(A) Average speed of the particle passing the windows may be equal if $W_1 < W_2 < W_3$

(B) Average speed of the particle passing the windows may be equal if $W_1 > W_2 > W_3$.

(C) If $W_1 = W_2 = W_3$, the change in speed of the particle while crossing the windows will satisfy $\Delta V_1 < \Delta V_2 < \Delta V_3$.

(D) If $W_1 = W_2 = W_3$, the time taken by particle to cross the windows will satisfy $t_1 < t_2 < t_3$.

6. The potential energy function of a particle is given by $U(r) = \frac{A}{2r^2} - \frac{B}{3r}$, where A and B are constants and r is the radial distance from the centre of the force. Choose the correct option(s).

(A) The equilibrium distance will be $r_0 = \frac{2A}{B}$

(B) The equilibrium distance will be $r_0 = \frac{3A}{B}$

(C) If the total energy of the particle is $\frac{B^2}{6A}$, then its radial velocity will vanish at $\frac{r_0}{3}$, where r_0 is the equilibrium distance.

(D) If the total energy of the particle is $\frac{B^2}{6A}$, then its radial velocity will vanish at $\frac{r_0}{2}$, where r_0 is the equilibrium distance.

7. Velocity of a particle of mass 2 kg changes from $\vec{v}_1 = -2\hat{i} - 2\hat{j}$ m/s to $\vec{v}_2 = (\hat{i} - \hat{j})$ m/s after colliding with a smooth plane surface.

(A) The angle made by the plane surface with the positive x-axis is $90° + \tan^{-1}\left(\frac{1}{3}\right)$

(B) The angle made by the plane surface with the positive x-axis is $\tan^{-1}\left(\frac{1}{3}\right)$

(C) The direction of change in momentum makes an angle $\tan^{-1}\left(\frac{1}{3}\right)$ with the positive x-axis.

(D) The direction of charge in momentum makes an angel $90° + \tan^{-1}\left(\frac{1}{3}\right)$ with the plane surface.

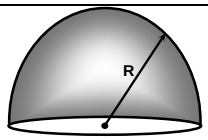
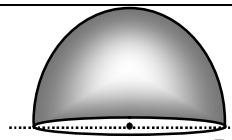
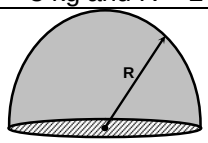
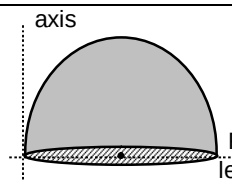
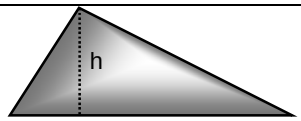
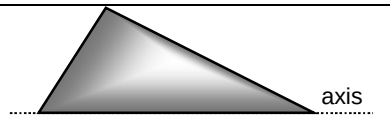
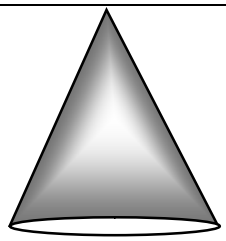
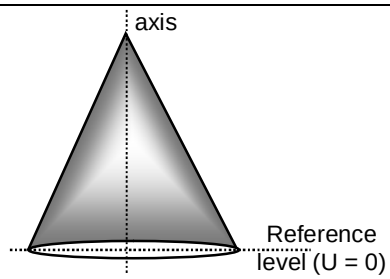
Space for Rough work

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 8, 9 and 10 by appropriately matching the information given in the three columns of the following table.

Match the following

Column 1 (Body)		**Column 2 (Axis and reference level)**		**Column 3 (magnitude of physical quantity in S.I. Unit)**	
(I)	 Uniform hollow hemispherical shell of M = 3 kg and R = 2 m	(i)		(P)	1
(II)	 Uniform solid hemisphere of M = $\frac{5}{7}$ kg and R = 3 m	(ii)		(Q)	2
(III)	 Uniform triangular plate of M = 9 kg and h = 3 m	(iii)		(R)	3
(IV)	 Uniform hollow cone of M = 2 kg, R = 1m and h = 3 m	(iv)		(S)	9

Space for Rough work

8. Distance of centre of mass (in m) of the body (shown in column-1) from the axis (shown in column-2)
(A) (I) (i) (Q) (B) (II) (ii) (S)
(C) (III) (iii) (P) (D) (IV) (iv) (R)

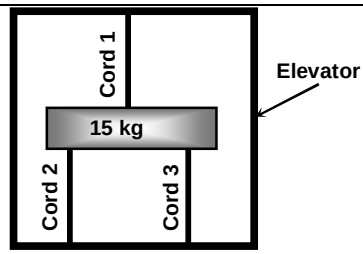
9. Moment of inertia of body (in kg-m^2) (shown in column –1) about the axis (shown in column –2)
(A) (I) (i) (S) (B) (II) (ii) (S)
(C) (III) (iii) (Q) (D) (IV) (iv) (R)

10. $\frac{\text{Gravitational potential energy } (U = 0, \text{ at the reference level})}{g}$ (in J-s^2/m) of the body (shown in column-1) with respect to the reference level (shown in column-2), where g is gravitation acceleration.
(A) (I) (i) (S) (B) (II)(ii) (R)
(C) (III) (iii) (P) (D) (IV) (iv) (Q)

Space for Rough work

Answer questions 11, 12 and 13 by appropriately matching the information given in the three columns of the following table.

A block of mass m = 15 kg is suspended in an elevator with the help of three identical light elastic cords (spring constant k = 100 N/m each) attached vertically. One of them, cord 1 is tied to the ceiling of the elevator and the other two cords 2 and 3 are tied to the elevator floor as shown in the figure. When the elevator is stationary the tension force in each of the lower cords is T = 7.5 N. Take g = 10 m/s^2. Now the elevator starts moving with given four accelerations shown in column I. Column II given the displacement of the block with respect to elevator when it is accelerating. Column III gives tension in the cords when elevator is accelerating.



Column 1 (acceleration of the elevator in different cases)		**Column 2** (displacement of the block with respect to the elevator upto new equilibrium position)		**Column 3** (Tension in the cord)	
(I)	1 m/s^2 upward	(i)	7.5 cm downward	(P)	2.5 N
(II)	1.5 m/s^2 upward	(ii)	2.5 cm downward	(Q)	5 N
(III)	2 m/s^2 upward	(iii)	15 cm downward	(R)	172.5 N
(IV)	33 m/s^2 downward	(iv)	5 cm downward	(S)	0 N

11. Tension in cord-1.
(A) (I) (i) (S)
(B) (II)(i) (R)
(C) (III) (ii) (P)
(D) (IV) (iii) (Q)

12. Tension in cord 2.
(A) (I) (iv) (P)
(B) (II)(i) (Q)
(C) (III) (iii) (R)
(D) (IV) (ii) (S)

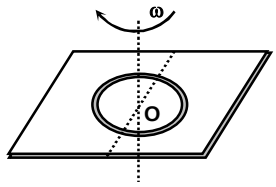
13. Tension in cord 3.
(A) (I) (i) (P)
(B) (II)(ii) (P)
(C) (III) (iii) (S)
(D) (IV) (iii) (R)

Space for Rough work

SECTION – C
(Single digit integer type)

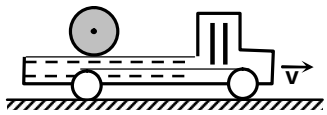
This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

14. A uniform ring of mass m is placed on a rough horizontal fixed surface as shown in the figure. The coefficient of friction between the left part of the ring and left part of the horizontal surface is $\mu_1 = 0.6\pi$ and between right half and the surface is $\mu_2 = 0.2\pi$. At the instant shown, now the ring has been imparted an angular velocity in clockwise sense in the figure shown. At this moment magnitude of acceleration of centre O of the ring (in m/s^2) is (take $g = 10\ m/s^2$)

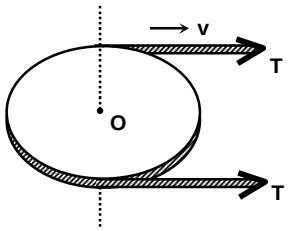


15. A power output from a certain experimental car design to be shaped like a cube is proportional to the mass m of the car. The force of air friction on the car is proportional to Av^2, where v is the speed of the car and A is the cross-sectional area. On a level surface the car has a maximum speed v_{max}. Assume that all versions of this design have the same density. Then v_{max} is proportional to $m^{1/C}$. Find C.

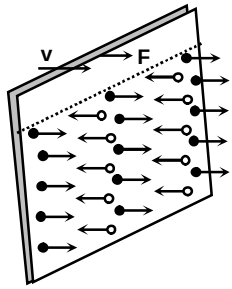
16. A solid cylinder rolls from the back of a large truck travelling at 10 m/s to the right. The cylinder is travelling horizontally at 8 m/s to the left relative to an observer in the truck. The ball lands on the roadway 1.25 m below its starting level. How far behind the truck does it land (in m)?



17. A flexible drive belt runs over a frictionless pulley as shown in figure. The pulley is rotating freely about the vertical axis passing through the centre O of the pulley. The vertical axis is fixed on the horizontal smooth surface. The mass per unit length of the drive belt is 1 kg/m and the tension in the drive belt 8 N. The speed of the drive belt is 2 m/s. Find the net normal force applied by the belt on the pulley in newton.



18. A thin heavy metal plate is being bombarded by a perpendicular beam of gas particles from both sides as shown in the figure. The solid dots are representing the molecules hitting from left side and the faint dots are the molecules hitting from right side. The mass of these gas particles is $m = 10^{-26}$ kg and velocity before hitting is $v_0 = 5$ m/s. Volume density of the gas particles on both sides is $n = 10^{25}$ per m^3. Each beam has an area $A = 1\ m^2$ and the collisions are perfectly elastic. What is the external force F (in newton) required to move the plate with a constant velocity v = 2 m/s.



Space for Rough work

Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **7** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. Pure $PCl_5(g)$ was introduced into an evacuated vessel and comes to equilibrium at 247°C and 2 atm pressure. At equilibrium the equilibrium mixture contains 40% by volume of $Cl_2(g)$

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

Which of the following statement(s) regarding the above equilibrium is/are correct?
(A) K_P for the equilibrium is 1.6 atm.
(B) K_C for the equilibrium is 0.071.
(C) Addition of inert gas to the equilibrium mixture at constant pressure increase the number of moles of PCl_3.
(D) Increase in pressure increases the number of moles of Cl_2.

20. If 0.53 $\overset{o}{A}$ is the Bohr's radius for the 1st orbit of hydrogen atom, the correct statement(s) in the light of wave mechanical model is/are
(A) The product of ψ^2 and $4\pi r^2$ increases till it reaches at the distance of 0.53 $\overset{o}{A}$.
(B) ψ^2 goes on increasing till it reaches at the distance of 0.53 $\overset{o}{A}$.
(C) ψ goes on decreasing till it reaches at the distance of 0.53 $\overset{o}{A}$.
(D) The nodal point for the 1s orbital is at 0.53 $\overset{o}{A}$.

21. The disproportionation reaction(s) among the following reactions is/are
(A) $2NO_2(g) + 2OH^- \longrightarrow NO_2^-(aq) + NO_3^-(aq) + H_2O(\ell)$
(B) $P_4(s) + 3OH^-(aq) + 3H_2O(\ell) \longrightarrow PH_3(g) + 3H_2PO_2^-(aq)$
(C) $Cl_2(g) + 2OH^-(aq) \longrightarrow Cl^-(aq) + ClO^-(aq) + H_2O(\ell)$
(D) $(CN)_2 + 2OH^-(aq) \longrightarrow CN^-(aq) + OCN^-(aq) + H_2O(\ell)$

22. The correct among the following statements is/are
(A) Calcium carbide on hydrolysis produces acetylene
(B) Calcium carbide when heated with N_2 at 1100°C forms calcium cyanamide
(C) Cyanamide ion is isoelectronic with CO_2
(D) Beryllium carbide on hydrolysis forms CH_4

Space for Rough work

23. Two equilibria are simultaneously existing in a vessel at 25°C

$NO(g) + NO_2(g) \rightleftharpoons N_2O_3(g)$ K_{P_1}

$2NO_2(g) \rightleftharpoons N_2O_4(g)$ $K_{P_2} = 8\ atm^{-1}$

Initially only NO(g) and NO_2(g) are present in 3 : 5 mole ratio. The total pressure at equilibrium is 5.5 atm and the partial pressure of NO_2 at equilibrium is 0.5 atm.
The incorrect statement(s) regarding the above equilibria is/are

(A) K_{P_1} for the equilibrium is 0.4 atm^{-1}.

(B) Partial pressure of N_2O_4 at equilibrium is 1.6 atm.

(C) Partial pressure of N_2O_3 at equilibrium is 2 atm.

(D) Partial pressure of NO at equilibrium is 2.5 atm.

24. The correct statement among the following is/are

(A) The B – F bond length of BF_3 is smaller than the B – F bond length of BF_4^-.

(B) Diborane on hydrolysis forms H_3BO_3.

(C) Borazine on hydrolysis produces N_3H.

(D) In $T\ell I_3$ oxidation state of $T\ell$ is +3.

25. 60 ml of 0.1 M NH_4OH is mixed with 40 ml of 0.1 M HCl. Which of the following statement(s) is/are INCORRECT about the resulting solution/mixture? (Given $pK_b(NH_4OH) = 4.74$, log2 = 0.3)

(A) The pH of the resulting solution is 5.04.

(B) The resulting solution is a basic buffer.

(C) The pOH of the resulting solution is 9.26.

(D) The resulting solution is acidic due to hydrolysis of NH_4Cl.

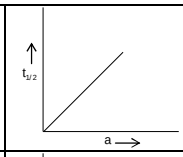
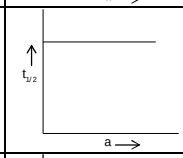
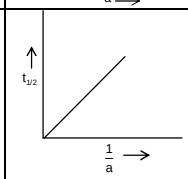
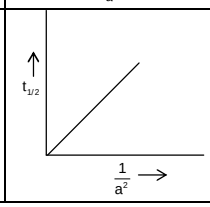
Space for Rough work

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 26, 27 and 28 by appropriately matching the information given in the three columns of the following table.

Columns 1 contains integrated rate equation, Column 2 contains half-life and Column 3 contain graph corresponding to different order reactions, 'a' is initial concentration of reactant and a – x is the concentration at time 't'.

Column 1		Column 2		Column 3	
(I)	$k=\frac{1}{t}\left(\frac{1}{a-x}-\frac{1}{a}\right)$	(i)	$t_{1/2}=\frac{a}{2k}$	(P)	
(II)	$k=\frac{x}{t}$	(ii)	$t_{1/2}=\frac{0.693}{k}$	(Q)	
(III)	$k=\frac{2.303}{t}\log\frac{a}{a-x}$	(iii)	$t_{1/2}=\frac{1}{ka}$	(R)	
(IV)	$k=\frac{1}{2t}\left[\frac{1}{(a-x)^2}-\frac{1}{a^2}\right]$	(iv)	$t_{1/2}=\frac{3}{2ka^2}$	(S)	

26. For the decomposition of HI on the surface of gold the only correct combination is
(A) (III) (ii) (R) (B) (II)(i) (P)
(C) (IV) (i) (P) (D) (I) (iii) (z)

27. For the reaction $N_2O_5 \longrightarrow 2NO_2+\frac{1}{2}O_2$, the only correct combination is
(A) (III) (ii) (Q) (B) (I)(iii) (P)
(C) (II) (i) (S) (D) (IV) (i) (P)

28. For the reaction $2A \longrightarrow Product$, the rate equation is
$r=k[A]^2$
Which of the following is the only correct combination for the reaction?
(A) (II) (iii) (P) (B) (II)(iii) (S)
(C) (III) (i) (S) (D) (I) (iii) (R)

Space for Rough work

Answer 29, 30 and 31 by appropriately matching the information given in the three columns of the following table.

Column 1 Hybridisation of central atom		**Column 2 Shape of molecule/ion**		**Column 3 No of lone pair in the central atom**	
(I)	sp^2	(i)	Linear	(P)	0
(II)	sp^3	(ii)	Square pyramidal	(Q)	1
(III)	sp^3d	(iii)	T-shaped	(R)	2
(IV)	sp^3d^2	(iv)	See-saw	(S)	3

29. For the ion BrF_5 the only correct combination is
(A) (III) (ii) (Q) (B) (IV)(ii) (Q)
(C) (III) (iv) (P) (D) (IV) (iv) (R)

30. For the ion $[ICl_2]^-$, the only correct combination is
(A) (III) (i) (S) (B) (II)(iii) (Q)
(C) (IV) (iv) (R) (D) (IV) (i) (R)

31. For the molecule $XeOF_2$, the only correct combination is
(A) (IV) (ii) (Q) (B) (IV)(iii) (Q)
(C) (III) (iv) (P) (D) (III) (iii) (R)

Space for Rough work

SECTION – C
(Single digit integer type)

This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

32. 4 mol of a mixture containing one mol each of $LiNO_3$, $NaNO_3$, $Ca(NO_3)_2$ and $Mg(NO_3)_2$ is decomposed by strongly heating. The total number of moles of $NO_2(g)$ evolved is (assuming complete decomposition of all the salts)

33. The velocity of an electron in the orbit of hydrogen atom is 5.47×10^5 m/sec. Total number of waves formed by the electron in one complete revolution is

34. How many of the following species are paramagnetic?
N_2^+, O_2^-, B_2, C_2, CN^-, NO^+, CO, H_2^-, He_2^+

35. 6 ml of 0.1 M CH_3COOH when mixed with x ml of 0.1 M NaOH, the pH of the resulting buffer solution was found to be 5.04. The value of x is (log2 = 0.3) $pK_{a(CH_3COOH)} = 4.74$.

36. For the reaction
$A(g) \rightleftharpoons B(g) + C(g)$
At 300 K, the average molecular weight of the equilibrium mixture is 83 g/mol. If atomic weight of A, B and C are 100, 60 and 40 gram/mol respectively, then the number of moles of 'C' present at equilibrium in a reaction starting with 10 moles of A is

Space for Rough work

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

This section contains **7** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. If α is the root of the equation $x - \tan x = 3$ where $\alpha \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; then which of the following is/are correct?, (where [.] denotes the greatest integer function and {.} fractional part function)

(A) $\lim\limits_{x\to\alpha^+}\left[\frac{\max(\tan x, \{x\})}{x-3}\right] = 1$

(B) $\lim\limits_{x\to\alpha^+}\left[\frac{\min(\tan x, \{x\})}{x-3}\right] = 1$

(C) $\lim\limits_{x\to\alpha^-}\left[\frac{\min(\tan x, \{x\})}{x-3}\right] = 0$

(D) $\lim\limits_{x\to\alpha^-}\left[\frac{\max(\tan x, \{x\})}{\tan x}\right] = 1$

38. Which of the following is/are incorrect?

(A) let f: R $\to$ R, such that $f(x) = 2x + \left[\frac{x(x^2-1)}{4(x^4-x^2+1)} + \frac{1}{8}\right] + [x] + \sin x \cos x$ then (where [.] denotes the greatest integer function) f is one-one onto

(B) let f: R $\to$ R, such that $f(x) = \frac{x^3+2x^2}{\sin x + 2}$ then f is one-one onto

(C) let f: R $\to$ [1, ∞) such that $f(x) = 2^{|x-1|^{\sqrt{2}}}$ then f(x) is one-one into

(D) let f: R – {0} $\to$ R such that f(x) = |x|ln|x| then f(x) is one-one onto

39. Let $S_n = \sum\limits_{r=1}^{n}\left(\frac{r^4+r^3n+r^2n^2+2n^4}{n^5}\right)$ and $T_n = \sum\limits_{r=0}^{n-1}\left(\frac{r^4+r^3n+r^2n^2+2n^4}{n^5}\right)$, n = 1, 2, 3, ... then

(A) $T_n > \frac{167}{60}$

(B) $T_n < \frac{167}{60}$

(C) $S_n > \frac{167}{60}$

(D) $S_n < \frac{167}{60}$

Space for rough work

40. Which of the following option(s) is/are incorrect?

(A) $\int_0^{\pi/8}\left|\frac{\sin(8nx)+\cos(8nx)}{x}\right|dx < \frac{2\sqrt{2}}{\pi}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right)$

(B) $\frac{2}{\ln 3} < \int_0^1 3^{x^{1/3}}dx < \int_0^1 4^{x^{1/3}}dx$

(C) $\int_0^{\pi/6}\sqrt{\sin x}\left(8-3\sqrt{\sin x}\right)dx > \frac{8\pi}{9}$

(D) $\frac{1}{4} < \int_{\pi/4}^{\pi/3}\frac{\tan x}{x}dx < \frac{1}{\sqrt{3}}$

41. The most general solution of the differential equation $\frac{x+y\frac{dy}{dx}}{x-y\frac{dx}{dy}} = \frac{2y^3}{x^5}\sin^2\left(x^2+y^2\right)$ is

(A) $-\frac{1}{2}\cot\left(x^2+y^2\right)-\frac{2(y/x)^4}{4}+c=0$ (B) $-\frac{1}{2}\cot\left(x^2+y^2\right)-\frac{2(y/x)^4}{4}+e^c=0$

(C) $-\frac{1}{2}\cot\left(x^2+y^2\right)-\frac{2(y/x)^4}{4}+\tan c=0$ (D) $\frac{1}{4}\tan\left(x^2+y^4\right)-\frac{2y^3}{x}+c=0$

42. Let $f(x) = 2x^3 - 3x^2 - x + \frac{3}{2}$ then which of the following is/are incorrect?

(A) $\int_{1/8}^{7/8} f\left(f(x)\right)dx = \frac{3}{4}$

(B) $y = \left(f(x)\right)^{3/5}$ is not differentiable at exactly 1 point

(C) $y = [f(x) + \lambda]$, ($\lambda \in$ prime number) is discontinuous at 3 points if $x \in [0, 1]$

(D) $f(f(x)) = 0$ has minimum 6 real roots

43. If $\int e^{x\sec^2 x-\tan x}\left(x\tan x-\frac{\sin 2x}{2}\right)dx = e^{f(x)}\cdot g(x)+c$ then which of the following is/are incorrect?

(A) f(0) is equal to = 1

(B) number of solution(s) of the equation $\left(g(x)\right)^{3/2} = \frac{1}{2\sqrt{2}}x^3$ is 3

(C) number of solution(s) of the equation $\left(g(x)\right)^{3/2} = \frac{1}{2\sqrt{2}}x^3$ is 1

(D) f(x) is an even function

Space for rough work

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 44, 45 and 46 by appropriately matching the information given in the three columns of the following table.

Column-1: real valued function; Column-2: continuity of the function; Column-3: differentiability of the function; Match the following Column(s)

Column–1	Column–2	Column–3
(I) $f(x) = \|\|x-6\| - \|x-8\|\| - \|x^2-4\| + 3x - \|x-7\|^3$	(i) continuous $\forall\ x \in R$	(P) not differentiable at 3 points
(II) $f(x) = (x^2-9)\|x^2+11x+24\| + \sin\|x-7\| + \cos\|x-4\| + (x-1)^{3/5}\sin(x-1)$	(ii) discontinuous at a single point only	(Q) not differentiable at 4 points
(III) $f(x) = \begin{cases} (x+1)^{3/5} - \frac{3\pi}{2} & : \ x < -1 \\ \left(x-\frac{1}{2}\right)\cos^{-1}\left(4x^3-3x\right) & : \ -1 \le x \le 1 \\ (x-1)^{5/3} & : \ 1 < x < 2 \end{cases}$	(iii) discontinuous at 2 points	(R) not differentiable at 2 point
(IV) $f(x) = \{\sin x\}\{\cos x\} + (\sin^3\pi\{x\})([x])$, $x \in [-1, 2\pi]$	(iv) discontinuous at 3 points	(S) not differentiable at 5 points

(Where [.] denotes the greatest integer function and {.} fractional part function)

44. Which of the following combination is correct?
(A) (I) (i) (R) (B) (III) (ii) (R)
(C) (IV) (iv) (P) (D) (I) (i) (Q)

45. Which of the following combination is correct?
(A) (II) (iii) (S) (B) (III) (i) (P)
(C) (II) (iii) (R) (D) (III) (i) (R)

46. Which of the following is correct?
(A) (I) (i) (S) (B) (IV) (i) (S)
(C) (II) (ii) (S) (D) (II) (ii) (P)

Space for rough work

Answer 47, 48 and 49 by appropriately matching the information given in the three columns of the following table.

Column-1: real valued function; Column-2: domain of the function; Column-3: range of the function.
Match the following Column(s) (where [.] denotes the greatest integer function and {.} denotes fractional part function)

Column–1	Column–2	Column–3
(I) $f_1(x)=\left[\log_{\sqrt{2}}\sqrt{\left\|(x-1)^3\right\|+\left\|(x-3)^3\right\|}\right]$	(i) number of integers in the domain of f(x) is 2	(P) set of all non positive integers $\cup$ {1}
(II) $f_2(x)=\left[\log_{\sqrt{2}}\left(\sin\left(\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)+\cos\left(\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)\right)\right]$	(ii) number of integers not in the domain of f(x) is 2	(Q) set of all non negative integers
(III) $f_3(x)=\left\{\frac{\left\|x+5\right\|-2}{x^2+10x+21}\right\}$	(iii) number of integers in the domain of f(x) is 3	(R) set of all positive integers
(IV) $f_4(x)=\left[\frac{1}{\sqrt{\ln\left(\cos^{-1}x\right)}}\right]$	(iv) number of integers in the domain of f(x) is infinite	(S) $\left(0,\frac{1}{2}\right]$

47. Which of the following combination is correct?
(A) (I) (iv) (P) (B) (I) (iv) (R)
(C) (II) (ii) (R) (D) (III) (i) (S)

48. Which of the following combination is correct?
(A) (IV) (i) (P) (B) (III) (ii) (S)
(C) (IV) (i) (Q) (D) (II) (iii) (S)

49. Which of the following combination is correct?
(A) (II) (iv) (P) (B) (IV) (i) (P)
(C) (II) (i) (P) (D) (IV) (i) (R)

Space for rough work

SECTION – C
(Single digit integer type)

This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

50. If the value of A for which the equation $\cot^3 A + \cot^2 A\,|\cot A + x| + |\cot^2 Ax + 1| = 1$ has not less than 6 different solutions which are integers are $[\cot^{-1}\alpha, \pi) \cup [\cot^{-1}\beta, \cot^{-1}\gamma]$, then $|\alpha + 30(\beta\gamma)^2|$ is equal to _____ (where $A \in (0, \pi)$)

51. Let f(x) be a differentiable and g(x) be a twice differentiable function such that $|f(x) - 1| \le 1$ and $f'(x) = g(x)$. If $f^2(3) + g^2(3) = 20$ then number of value(s) of $c \in (0, 4)$ such that $g(c)g''(c) > 0$ is/are _____

52. Let f(x) be a non-constant thrice differential function defined on $(-\infty, \infty)$ such that $f\left(\frac{x+13}{2}\right) = f\left(\frac{3-x}{2}\right)$ and $f'(0) = f'\left(\frac{1}{2}\right) = f'(2) = f'(3) = f'\left(\frac{9}{2}\right) = 0$ then the minimum number of zeroes of $h(x) = (f''(x))^2 + f'(x)f'''(x)$ in the interval [0, 9] is 2k then k is equal to _____

53. If $\forall\ h \in R - \{0\}$, two distinct tangents can be drawn from the points (2 + h, 3h – 1) to the curve $y = x^3 - 6x^2 - a + bx$ then $\frac{a}{b}$ is equal to _____

54. Let $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{\lambda k^4 + 2k^3 + k^2 + k + 1}{3n^5 + n^2 + n + 5k} = \frac{1}{3}$ then λ is equal to _____

Space for rough work