FIITJEE **JEE(Advanced)-2018**

ALL INDIA TEST SERIES

ANSWERS, HINTS & SOLUTIONS

PART TEST – I

PAPER-2

ANSWERS KEY

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	**D**	24.	**B**	47.	**B**
2.	**A**	25.	**C**	48.	**A**
3.	**B**	26.	**C**	49.	**A**
4.	**A, D**	27.	**B, C, D**	50.	**A, B, D**
5.	**C**	28.	**A, C**	51.	**A, B**
6.	**A, C**	29.	**A**	52.	**B, C**
7.	**B, C, D**	30.	**A, D**	53.	**D**
8.	**B, D**	31.	**B, C, D**	54.	**B, D**
9.	**B**	32.	**A**	55.	**A**
10.	**B**	33.	**D**	56.	**A**
11.	**5**	34.	**5**	57.	**2**
12.	**9**	35.	**7**	58.	**7**
13.	**8**	36.	**8**	59.	**1**
14.	**4**	37.	**4**	60.	**4**
15.	**6**	38.	**2**	61.	**3**
16.	**5**	39.	**4**	62.	**7**
17.	**2**	40.	**3**	63.	**0**
18.	**6**	41.	**3**	64.	**9**
19.	**7**	42.	**7**	65.	**1**
20.	**3**	43.	**3**	66.	**7**
21.	**00006.25**	44.	**205.02**	67.	**00000.12**
22.	**00006.40**	45.	**28.88**	68.	**00000.21**
23.	**00018.75**	46.	**8.34**	69.	**00000.15**

Physics

PART – I

SECTION – A

1. $D = \sqrt{\dfrac{2H}{g}}\sqrt{2g(2R)}$

2. Moment of inertia of combined system about y-axis

$$= \frac{4m\left(\frac{R}{2}\right)^2}{4} + \frac{mR^2}{4} + \frac{mR^2}{2} + mR^2 = 2mR^2$$

Conservation of angular momentum along y-axis

$$mv_0\frac{R}{2} = 2mR^2\omega$$

$$\omega = \frac{v_0}{4R}$$

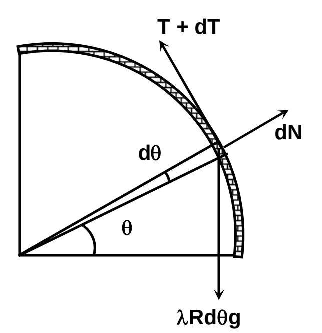
3. $\vec{f}_k$ on the block B = $\mu mg\left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}\right)$

$$P_{f_k} = \mu mg\left[\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{k}}{\sqrt{2}}\right]\cdot(v_0\hat{k})$$

$$= -\frac{\mu mgv_0}{\sqrt{2}}$$

$$\frac{dQ}{dt} = \mu mgv_0\frac{2}{\sqrt{2}} = \sqrt{2}\mu mgv_0$$

4. For equilibrium



$$\lambda Rg\int_0^{\frac{\pi}{2}}\cos\theta d\theta = \mu\lambda Rg\int_0^{\frac{\pi}{2}}\sin\theta d\theta$$

$\therefore$ $\mu = 1$

At the position of maximum tension in the rope

$$\lambda Rd\theta g\cos\theta = \mu(\lambda Rd\theta g\sin\theta)$$

$\therefore$ $\theta = 45°$

At any θ,

$$dT = \lambda Rd\theta g\cos\theta - \mu\lambda Rd\theta g\sin\theta$$

$$\int_0^{T_{max}} dT = \lambda Rg\int_0^{\frac{\pi}{4}}(\cos\theta - \sin\theta)d\theta$$

$$T_{max} = \lambda Rg[\sin\theta + \cos\theta]_0^{\frac{\pi}{4}} = \lambda Rg\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] = \lambda Rg\left(\sqrt{2} - 1\right)$$

5. Acceleration of particle P and Q along the incline is same.

Acceleration of particle P perpendicular to the incline plane is 10 cos 60° = 5 m/s^2

So, $v_P - 5(2) = 0$

$\therefore$ $v_P = 10$ m/s

6. $\omega R = u = \text{constant}$

$$\omega\frac{dR}{dt} + R\frac{d\omega}{dt} = 0$$

$$\therefore\ \frac{d\omega}{dt} = -\frac{\omega}{R}\frac{dR}{dt}$$

In time dt radius decreases by dR

$-2\pi R dR = u d dt$

$$\therefore\ \frac{dR}{dt} = -\frac{ud}{2\pi R}$$

$$\frac{d\omega}{dt} = \alpha = \frac{\omega}{R}\frac{ud}{2\pi R}$$

$$\alpha = \frac{u^2 d}{2\pi R^3}$$

$$\tau = \frac{Iu^2 d}{2\pi R^3}$$

$$\therefore\ F = \frac{Iu^2 d}{2\pi R^4}$$

7. For pure rolling motion, static friction would act in the forward direction on the sphere. Work done by static friction is change in the kinetic energy of the sphere. Work done by static friction on the complete sphere + Plank system is zero.

8. F applied by external agent = Weight + thrust force = $\lambda xg + \lambda v^2$

Energy lost in the complete lift = $W_F - \Delta K.E. - \Delta U = \dfrac{\lambda \ell v^2}{2}$

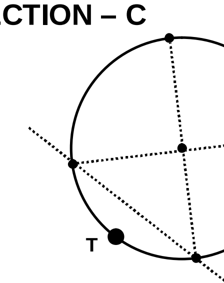
9. For perfectly in elastic collision both blocks have same velocity after collision.

10. $$e = \frac{\frac{4}{3} - \frac{2}{3}}{2} = \frac{1}{3}$$

SECTION – C

11. $$2g\left(R\frac{1}{\sqrt{2}}\right) \ge mg\left(R - \frac{R}{\sqrt{2}}\right)$$

$$m \le \frac{2}{\sqrt{2}-1} = 5\,\text{kg}$$



12. $$W = \int_0^{\ell} \mu 2\pi r x dx P$$

$= \pi\mu r P \ell^2$

$= \pi(0.3)(3 \times 10^{-2})\dfrac{10^5}{\pi} \times 10^{-2} = 9\ \text{J}$

13. $v = \sqrt{v_0^2 + 2gy}$

and v sin θ = v_0 (as given in question)

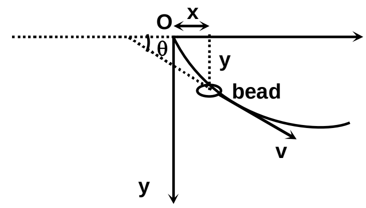
$\therefore \sin\theta = \frac{v_0}{\sqrt{v_0^2 + 2gy}}$

$\therefore \tan\theta = \frac{v_0}{\sqrt{2gy}}$

$\frac{dy}{dx} = \frac{v_0}{\sqrt{2gy}}$

$\therefore y = \frac{(3gv_0x)^{2/3}}{2g}$

$\therefore$ a + b + c = 8

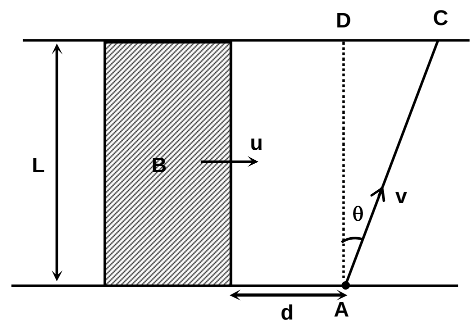


14. Time to cross, $t = \frac{L}{v\cos\theta}$

$\therefore ut = d + L\tan\theta = \frac{uL}{v\cos\theta}$

$\therefore v = \frac{uL}{d\cos\theta + L\sin\theta}$

$\therefore v_{min} = \frac{uL}{\sqrt{d^2 + L^2}} = 4$ m/s



15. If they meet a height h after time T of the projection of the second.

Then, $h = u(T) - \frac{1}{2}g(T)^2 = v(T-t) - \frac{1}{2}g(T-t)^2$ …(i)

$T = \frac{5t^2 + 3t}{10t - 2}$

For minimum T, $\frac{dT}{dt} = 0$

$50t^2 - 20t - 6 = 0$

$\Rightarrow t = 0.6 = \frac{6}{g}$

16. For car + trailer system

$P_{max} = (3mg\sin\theta + 3kmv^2)v$

For car only

$P_{max} = (mg\sin\theta + 4kmv^2)2v$

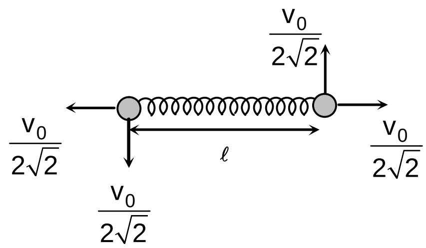
$\therefore (3mg\sin\theta + 3kmv^2)v = (mg\sin\theta + 4kmv^2)2v$

$\therefore k = \frac{g\sin\theta}{5v^2}$

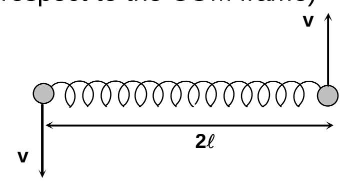
For down the incline $kmv_t^2 = mg\sin\theta$

$\therefore v_t^2 = 5v^2$

17. At the starting (with respect to COM frame)



At the instant of maximum elongation (with respect to the COM frame)



Conservation of angular momentum with respect to centre of mass (COM) frame

$$2m\frac{v_0}{2\sqrt{2}}\frac{\ell}{2} = 2mv\ell$$

$$\therefore\ v = \frac{v_0}{4\sqrt{2}}$$

From conservation of energy with respect to centre of mass frame

$$2\frac{1}{2}m\left(\frac{v_0^2}{8}+\frac{v_0^2}{8}\right) - 2\frac{1}{2}m\frac{v_0^2}{32} = U_{spring}$$

$$\therefore\ U_{spring} = mv_0^2\left(\frac{1}{4}-\frac{1}{32}\right) = \frac{7mv_0^2}{32} = 2\text{ Joule}$$

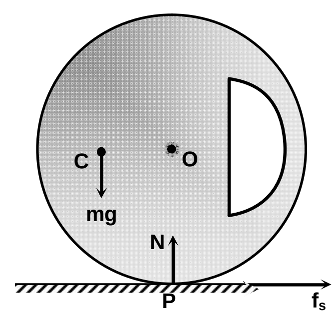
18. considering torque of real and pseudo force in the frame of P,

$$\tau = mg\frac{R}{2} + m\omega^2 R\frac{R}{2}$$

$$= mg\frac{R}{2} + m\frac{g}{R}R\frac{R}{2} = mgR$$

$$I_P = m\left(\frac{R}{2}\right)^2 + m\left(R^2+\frac{R^2}{4}\right) = \frac{3}{2}mR^2$$

$$\therefore\ \alpha = \frac{2mgR}{3mR^2} = \frac{2g}{3R} = 6\ \text{rad/s}^2$$



19. $\mu_s g = \frac{dv}{dt}$ at t = 1 sec

$\therefore\ \mu_s = 0.4$

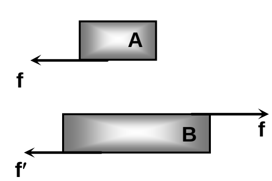
At t = 3 sec velocities of plank and block are equal.

$\therefore\ 2 + \mu_k g(2) = v$ at t = 3 sec = 8

$\therefore\ \mu_k = 0.3$

$\therefore$ s + k = 4 + 3 = 7

20. Since $\frac{\mu}{4} < \frac{\mu}{2} < \mu$, the upper block will move faster than the middle block and hence force of friction on upper block is towards left



$$\therefore\ f = \frac{\mu}{4}mg$$

$$\therefore\ a_A = \frac{\mu g}{4}\ \text{leftward}$$

$f' = \frac{\mu}{2}\left(m + \frac{m}{2}\right)g = \frac{3}{4}\mu mg$

$\therefore\ a_B = \mu g$ (leftward)

$\therefore\ a_{A/B} = \frac{3}{4}\mu g$ (rightward)

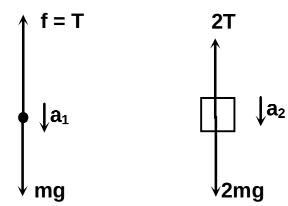
For sliding down the block B, A has to move a distance $\frac{3L}{8}$ relative to block B.

$\frac{3L}{8} = \frac{1}{2}\frac{3}{4}\mu g t^2$

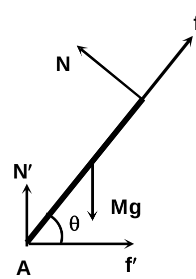
$\therefore\ t = \sqrt{\frac{L}{\mu g}} = 3\,\text{sec.}$

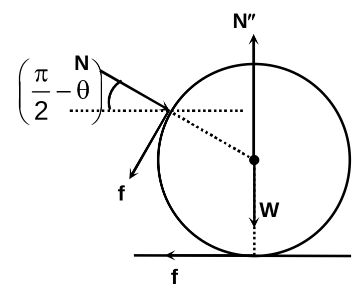
SECTION – D

21. $mg - T = ma_1$
$2mg - 2T = 2ma_2$
So, $a_1 = a_2$
Relative acceleration of bead with respect to end = 3a
$\therefore$ displacement of block x = $\frac{1}{2}at^2 = \frac{\ell}{3}$ = 6.25 m

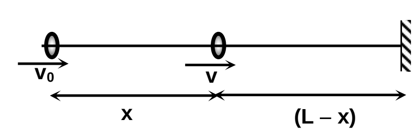


22. $\tau_A = 0$
$N = \frac{Mg}{2}\cos\theta$
For disc, N sin θ = f + f cos θ
$\therefore\quad f = \frac{N\sin\theta}{1+\cos\theta} = \frac{Mg\sin\theta\cos\theta}{2(1+\cos\theta)}$
$M = \lambda L$ and $L = \frac{R}{\tan\frac{\theta}{2}}$
$f = \frac{1}{2}\lambda gR\cos\theta$
$= \frac{1}{2}\times 8\times 10\times\frac{2}{10}\times\frac{4}{5}$
$= \frac{32}{5} = 6.40$ N





23. $v\frac{dv}{dx} = -k(L-x)$, where k is a constant.
$\int_{v_0}^{v_0/2} v\,dv = -k\int_0^L (L-x)dx$
$\Rightarrow \frac{3}{4}v_0^2 = KL^2$
So initial retardation = KL = $\frac{3}{4}\frac{v_0^2}{L} = \frac{3}{4}\times 25 = 18.75\ \text{m/s}^2$



Chemistry

PART – II

SECTION – A

$$2NH_4OH + H_2SO_4 \longrightarrow (NH_4)_2SO_4 + 2H_2O$$

24. Initial 1 m mol 0.4 m mol 0

0.2 m mol 0 0.4 m mol

$$[NH_4^+] = \frac{2\times 0.4}{14}, [NH_4OH] = \frac{0.2}{14}$$

$$\therefore pOH = pK_b + \log\frac{0.8/14}{0.2/14}$$

= 4.76 + log4 = 4.76 + 0.6

= 5.36

pH = 14 - 5.36 = 8.64

25. $\frac{E_{a_1}}{T_1} = \frac{E_{a_2}}{T_2}$

$\frac{200}{T_1} = \frac{100}{298}$

T_1 = 596 K = 323^{o}C

26. $2BF_3 + 6NaH \xrightarrow{180^oC} B_2H_6 + 6NaF$

27. P – F bond length is less than P – Cl bond length

P – F 159.6 pm P – Cl 200.5 pm.

28. (i) Li has stronger reducing character than Na in aqueous solution.

(ii) Li_2CO_3 is thermally less stable than Na_2CO_3.

29. Pyrosilicates have general formula $(Si_2O_7)^{6-}$.

30. (i) Formula of borax is $Na_2[B_4O_5(OH)_4].8H_2O$.

(ii) Aqueous solution of borax is weakly alkaline.

31. $K_P = P_{NH_3} \times P_{H_2S}$

Addition of NH_4HS(s) at equilibrium does not change the concentration of NH_3(g).

32. $[Ag^+] = \sqrt{\frac{K_{sp}Ag_2CO_3}{[CO_3^{2-}]}} = \sqrt{\frac{4\times 10^{-12}}{10^{-2}}} = 2\times 10^{-5}$ M

33. $[Cl^-] = \frac{K_{sp}AgCl}{[Ag^+]} = \frac{1.8\times 10^{-10}}{2\times 10^{-5}}$

= 9 × 10^{-6} M.

SECTION – C

34. $\lambda = \frac{2\pi r_n}{n}$

$$\lambda = \frac{2\times 3.14\times 0.529\times 10^{-10}\times n^2}{n}$$
$$1.67\times 10^{-9} = 2\times 3.14\times 0.529\times 10^{-10}\times n$$
$$\therefore n = 5$$

35. $2Cr(OH)_3 + IO_3^- + 4OH^- \longrightarrow 2CrO_4^{2-} + I^- + 5H_2O$

x = 2, y = 1, z = 4

$\therefore x + y + z = 2 + 1 + 4 = 7$

36.
$$\begin{array}{lll} A(g) + 3B(g) & \rightleftharpoons & 4C(g) \\ a - x \quad a - 3x & & 4x \\ a - x = 4x \quad a = 5x & & \end{array}$$

[A] = a – x = 5x – x = 4x

[B] = a – 3x = 5x – 3x = 2x

[C] = 4x

$$K_c = \frac{[C]^4}{[A]\times[B]^3}$$
$$= \frac{(4x)^4}{4x\times(2x)^3}$$
= 8

37.
$$\begin{array}{lll} CO_2(g) + C(s) & \rightleftharpoons & 2CO(g) \\ 2 - x & & 2x \end{array}$$
$\therefore 2 - x + 2x = 3$

x = 1

$P_{CO_2} = 2 - 1 = 1$ atm, $P_{CO} = 2x = 2\times 1 = 2$ atm

$$K_p = \frac{(P_{CO})^2}{P_{CO_2}} = \frac{(2)^2}{1} = 4$$

38. $\dfrac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\dfrac{P_2}{P_1}\right)^{n-1}$ n – 1 = 1

$\dfrac{10}{5} = \left(\dfrac{200}{100}\right)^{n-1}$ n = 2

$2 = 2^{n-1}$

39. Boron Nitride, Boric acid, Beryllium hydride, Graphite.

40. (i) 10^{-8} M HCl

(ii) 0.01 M solution of NH_4Cl

(iii) 0.01 M solutin of $B(OH)_3$

41. $[CH_3COOH] = C_1 = \dfrac{200\times 1}{400} = 0.5$

$[HCOOH] = C_2 = \dfrac{200\times 0.1}{400} = 0.05$

$$\left[H^+\right] = \sqrt{K_{a_1}C_1 + K_{a_2}C_2}$$
$$= \sqrt{\left(10^{-6} \times 0.5\right) + \left(10^{-5} \times 0.05\right)}$$
$= 10^{-3}$
pH = - log 10^{-3} = 3

42. The following molecules/ion are planar
$CO_3^{2-}, NO_3^-, XeF_4, I_3^-, COCl_2, SO_3, I_2Cl_6$ (solid)

43.
$$Al(OH)_3 + NaOH \longrightarrow \underset{(Soluble)}{NaAlO_2} + 2H_2O$$
$$Zn(OH)_2 + 2NaOH \longrightarrow \underset{(Soluble)}{Na_2ZnO_2} + 2H_2O$$
$$Sn(OH)_2 + 2NaOH \longrightarrow \underset{(Soluble)}{Na_2SnO_2} + 2H_2O$$

SECTION – D

44. For shortest wavelength $n_2 = \infty$
$$\frac{1}{\lambda_{min}} = 4 \times R_H\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$$
$$\frac{1}{\lambda_{min}} = \frac{4}{9}R_H$$
$$\lambda_{min} = \frac{9}{4} \times \frac{1}{R_H}\text{nm}$$
= 205.02 nm

45.
$$S_2Cl_4(g) \rightleftharpoons 2SCl_2(g)$$

	$S_2Cl_4(g)$	$2SCl_2(g)$
Initial	4	0
At equilibrium	0.2	7.6

$$\therefore [S_2Cl_4] = \frac{0.2}{10} \qquad [SCl_2] = \frac{7.6}{10}$$
$$K_c = \frac{[SCl_2]^2}{[S_2Cl_4]}$$
$$= \frac{\frac{7.6}{10} \times \frac{7.6}{10}}{\frac{0.2}{10}}$$
= 28.88

46. pOH at $\frac{1}{4}$th equivalence point = 14 – 9.3 = 4.7

At $\frac{1}{4}$th equivalence point
$$pOH = pK_b + \log\frac{[Salt]}{[Base]}$$
$$4.7 = pK_b + \log\frac{0.25x}{0.75x}$$

$$4.7 = pK_b + \log\frac{1}{3}$$

$$pK_b = 4.7 + \log 3$$

At $\frac{3}{4}$th equivalence point

$$pOH = pK_b + \log\frac{[\text{Salt}]}{[\text{Base}]}$$

$$pOH = 4.7 + \log 3 + \log\frac{0.75x}{0.25x}$$

$$pOH = 4.7 + \log 3 + \log 3$$

$$pOH = 4.7 + 2 \times \log 3$$

$$pOH = 4.7 + 2 \times 0.48$$

$$pOH = 5.66$$

pH = 14 – 5.66 = 8.34

Mathematics

PART – III

SECTION – A

47. Put $x = \frac{1}{1-t}$ in I_2 and $x = 1 - \frac{1}{t}$ in I_3

We get $\ell = \int_{-20}^{-10} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 \left(1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right) dx$

Let $u = \frac{x^3 - 3x + 1}{x(x-1)}$ then $\ell = \int \frac{du}{u^2}$

48. $\frac{x^2 dy - 2xy dx}{x^4} = \frac{x^3 y^2 (y dx + x dy)}{x^4}$

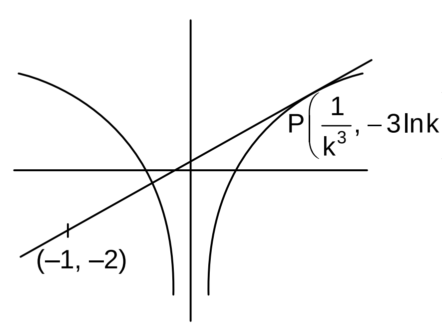
$$\frac{d\left(\frac{y}{x^2}\right)}{\frac{y}{x^2}} = xy\,d(xy)$$

$$\Rightarrow \ln\left(\frac{y}{x^2}\right) = \frac{(xy)^2}{2} + c$$

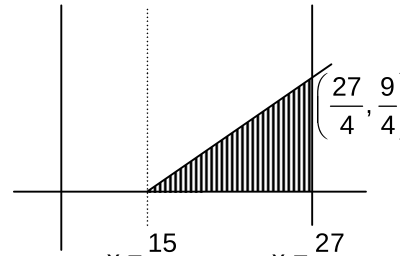
49. $\frac{-3\ln k + 2}{\frac{1}{k^3} + 1} = k^3$

So, $f(k) = k^3 + 3\ln k - 1$

$$f\left(\frac{1}{\sqrt{e}}\right) f(e) < 0$$



50. $$y' = \frac{3}{4}\left(\frac{4y^2+5+\frac{4x}{3}-5}{4y^2+5}\right) = \frac{3}{4}+\frac{1}{4}\left(\frac{4x-15}{4y^2+5}\right)$$



$y' \geq \frac{3}{4}$ for $4x - 15 \geq 0$

$$\int_{15/4}^{27/4} f(x)dx \geq \frac{1}{2}\times\frac{12}{4}\times\frac{9}{4}$$

51. $$\frac{\left((x^2-1)+4(y^2-1)\right)x}{\left(4(x^2-1)-3(y^2-1)\right)y} = \frac{dy}{dx}$$

Let $y^2 - 1 = v(x^2 - 1)$

$$\Rightarrow \frac{6v^2+2}{4-3v} = \left(\frac{x^2-1}{x}\right)\frac{dv}{dx}$$

$$\Rightarrow \int \frac{4-3v}{6v^2+2}dv = \int \frac{x}{x^2-1}dx$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\left(\frac{y^2-1}{x^2-1}\right)\right) - \frac{1}{4}\ln\left(6\left(\frac{y^2-1}{x^2-1}\right)^2+2\right) - \ln\sqrt{|x^2-1|} + c = 0$$

52. (A) $2C_1(2 \times 2 - 1)^5$
(B) $(2 \times 2 - 1)^5 \times 1$
(C) $(2 \times 2 - 1)^4 \times 1$
(D) $(2 \times 2 - 1)^2 = 9$

53. Let $g(x) = f(x) - x^2 - 1$
$g(1) = g(2) = 0$
So; $g'(C_1) = 0$ for at least one $C_1 \in (1, 2)$
Similarly $g'(C_2) = 0$ for at least one $C_2 \in (2, 3)$
Using LMVT on $x \in (C_1, C_2)$

$$g''(C) = \frac{g'(C_2)-g'(C_1)}{C_2-C_1} = 0$$

$f''(C_3) = 2$

54. Let $x = \sqrt{3}$
$f\left(\sqrt{3}\right) = 0$
$\because \sqrt{3} = 1.732050807.....$

As the decimal part increases then in the expression $\frac{p}{q}$, q becomes very large

So, $\frac{2}{2q^3 - q^2 + q + \sin^2 q + 5} \to 0$

Hence, $\lim_{x\to\sqrt{3}} f(x) = 0$

Thus, f(x) is continuous at each irrational

55.-56. $-7 \le \frac{\left(\sqrt{193}-1\right)}{2}\cos y + \cos\left(y+\frac{\pi}{3}\right) \le 7$

So; $g(x) = \begin{cases} |x+7| & ; \quad x \ge 0 \\ |x-7| & ; \quad x < 0 \end{cases}$

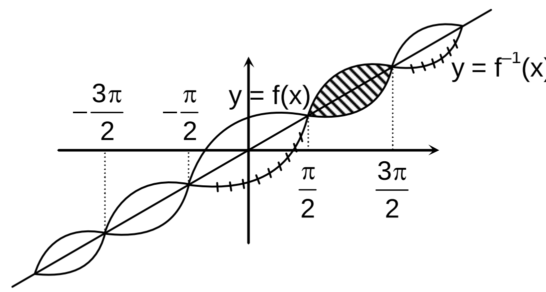
So; $g(x)_{min} = 7$

So; $f(x) = x + \cos x$

55. $A = 3\times 2\int_{\pi/2}^{3\pi/2}\left(x-\left(x+\cos x\right)dx\right)$

$= -3\times 2\left(\sin x\right)_{\pi/2}^{3\pi/2}$

$= -6(-1-1) = 12$



56. $2^x = x^2$ intersect at 3 points

SECTION – C

57. $g(x) = \begin{cases} x & : \quad 0 \le x \le 1 \\ 1 & : \quad 1 \le x \le 2 \\ -x+3 & : \quad x > 2 \end{cases}$

58. Q(x) is an increasing function

So, $\int_{e^3}^{e^6} Q'(x)dx \le \int_{e^3}^{e^6}\frac{2}{x}dx$

$Q(e^6) \le 7$

59. $\frac{1+x^{n+1}}{1+x^n} = 1+\frac{x^n(x-1)}{1+x^n}$

So, $\ln\left(\prod_{n=0}^{N}\left(1+\frac{x^n(x-1)}{1+x^n}\right)^{x^n}\right) = \sum_{n=0}^{N} x^n \ln\left(1+\frac{x^n(x-1)}{1+x^n}\right)$

So, $(x-1)\left(\frac{1}{1-x^2}-\frac{1}{1-x^3}+.....+\frac{1}{1-x^{2k}}\right) \le \sum_{n=0}^{\infty} x^n\left(\frac{x^n(n-1)}{1+x^n}\right)$

$\le (x-1)\left(\frac{1}{1-x^2}-\frac{1}{1-x^3}+.....-\frac{1}{1-x^{2k+1}}\right)$

So, limit = $\frac{2}{e}$

60. For least area $\lambda = 0$

So; $A_{min} = 2\int_0^4\left(12-\left(x^2-4\right)\right)dx = \frac{256}{3}$

61. $\tan^2 t \geq \frac{\theta^2 - \sin\theta^2}{\tan\theta^2 - \sin\theta^2} \forall \theta \in \left(0, \frac{\pi}{2}\right)$

So; $\tan^2 t \geq \left(\frac{\theta^2 - \sin\theta^2}{\tan\theta^2 - \sin\theta^2}\right)_{max} \forall \theta \in \left(0, \frac{\pi}{2}\right)$

Since; in $\left(0, \frac{\pi}{2}\right)$: $\tan \theta^2 > \theta^2$ and the same is subtracted from N^{-r} and D^{-r} both

So; maximum value occurs at $\theta \to O^+$

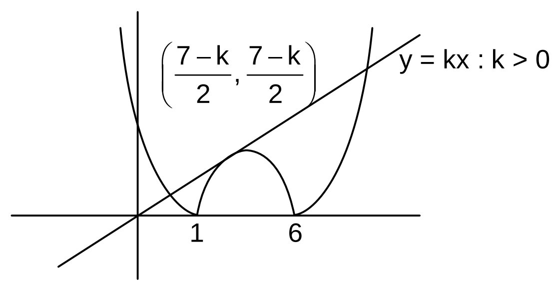
Therefore $\tan^2 t \geq \frac{1}{3}$; $t \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

62. So; $-2x + 7 = k$

$\Rightarrow x = \frac{7-k}{2}$

Also $-\frac{(7-k)^2}{4} + 7\left(\frac{7-k}{2}\right) - 6 = k\left(\frac{7-k}{2}\right)$

$\Rightarrow k = 7 + \sqrt{24}$, $7 - \sqrt{24}$ (Rejected)



So: $k \in \left[7 - \sqrt{24}, \infty\right)$

Similarly: consider case-II if k < 0 then $k \in \left(-\infty, -\left(7 + \sqrt{24}\right)\right]$

Second Method

Clearly: $m_1 < k < k_2$

For slope of ON

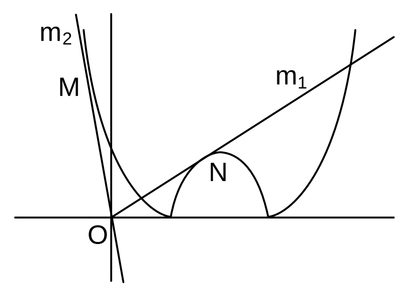
$-(x^2 - 7x + 6) = kx$

Must have equal roots

For slope of OM

$x^2 - 7x + 6 = kx$

Must have equal roots



63. $g(f(x)) = x$

So; $g'(f(x)) \cdot f'(x) = 1$

$g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$

So; $g'(3) = \frac{1}{23}$ and $g''(3) = \frac{-43}{5(23)^3}$

64. $f^{-1}(x) = [x] + \{x\}^{1/3}$

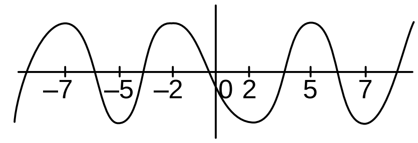
So, $\int_2^{9/2} [x]dx + \int_2^{9/2} \{x\}^{1/3}dx = \frac{17}{2} + \frac{3}{2^{10/3}}$

65. Let $f(x) = \int_0^x \frac{t^8 + 1}{t^8 + t^2 + 1}dt - 3x + 2$

f(0) = 2, f(1) = negative

So; f(x) has one root in [0, 1]

66. f is an odd function
So; f(7) = –7, f(5) = 5, f(2) = 3
$x \to -\infty$, $f(x) \to \infty$
$x \to \infty$, $f(x) \to \infty$



SECTION – D

67. Let $1 - x^3 = t$

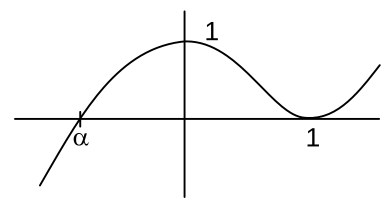
$$3I_1 = \int_0^1 \left(1 - t^{\sqrt{2}}\right)^{\sqrt{3}} dt \text{ and } 3I_2 = \int_0^1 \left(1 - t^{\sqrt{2}}\right)^{\sqrt{3}+1} \cdot 1 \cdot dt$$

$$= \int_0^1 \left(\sqrt{3}+1\right)\sqrt{2}\left(1 - t^{\sqrt{2}}\right)^{\sqrt{3}} \left(1 - t^{\sqrt{2}} - 1\right) dt \text{ (Using integration by parts)}$$

$$3I_2 = -\left(\sqrt{3}+1\right)\sqrt{2} \cdot 3I_2 + \left(\sqrt{3}+1\right)\sqrt{2} \cdot 3I_1$$

So, $\dfrac{I_1}{I_2} - \dfrac{\sqrt{3}-1}{2\sqrt{2}} + 0.2 = 1 + \dfrac{\sqrt{3}-1}{2\sqrt{2}} - \dfrac{\sqrt{3}-1}{2\sqrt{2}} + 0.2 = 1.2$

68. Roots of the f(x) = 0 are $\alpha \in (-1, 0)$ and 1
then f(f(x)) = 0 (where f(x) = t)
$f(t) = 0 \Rightarrow t = \alpha, 1$
So; total number of distinct real solutions = 2 + 1 = 3



69. $f''(x) = (x-1)^5(x+2)^6(195x^2 + 153x + 30)$
$f''(x) = 0$ if $x = 1, -2, x_1, x_2$
Clearly: there are 3 inflexion points