FIITJEE

JEE(Main)-2018

ALL INDIA TEST SERIES

ANSWERS, HINTS & SOLUTIONS

PART TEST – I
(Main)

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	A	31.	C	61.	D
2.	C	32.	D	62.	C
3.	C	33.	D	63.	D
4.	A	34.	C	64.	C
5.	D	35.	C	65.	C
6.	C	36.	D	66.	B
7.	C	37.	A	67.	C
8.	C	38.	B	68.	D
9.	D	39.	C	69.	C
10.	A	40.	B	70.	A
11.	B	41.	A	71.	D
12.	D	42.	C	72.	A
13.	A	43.	D	73.	B
14.	A	44.	C	74.	B
15.	A	45.	C	75.	C
16.	A	46.	D	76.	D
17.	B	47.	D	77.	A
18.	A	48.	D	78.	C
19.	B	49.	A	79.	D
20.	C	50.	A	80.	A
21.	C	51.	A	81.	D
22.	C	52.	D	82.	D
23.	B	53.	A	83.	B
24.	D	54.	A	84.	C
25.	C	55.	D	85.	D
26.	C	56.	A	86.	D
27.	B	57.	B	87.	D
28.	A	58.	B	88.	C
29.	D	59.	A	89.	A
30.	A	60.	B	90.	B

Physics

PART – I

SECTION – A

1. $v = \dfrac{dx}{dt}$

$\Rightarrow dt = \dfrac{dx}{v} = \dfrac{xdx}{a}$

$\Rightarrow \displaystyle\int_0^t dt = \frac{1}{a}\int_{x_1}^{x_2} xdx$

$\Rightarrow t = \dfrac{x_2^2 - x_1^2}{2a}$

2. Weight is non impulsive force during explosion.

3. Basic concept of direction

4. Conservation of momentum for (M +m)

$2mv_0 = Mv_1$ …(i)

Newton's second law :

$v_0 \cos 30° = v_1 \cos 30° + v_2 \cos 60°$

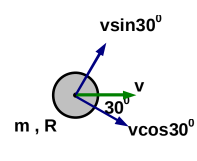
$\sqrt{3}v_0 = \sqrt{3}v_1 + v_2$ …(ii)

For 'm': $v_0 \sin 30° = v_2 \sin 60°$ …(iii)

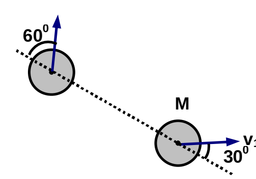
$\Rightarrow v_2 = \dfrac{v_0}{\sqrt{3}}$, $v_1 = \dfrac{2}{3}v_0$

from (i) $\dfrac{2mv_0}{M} = \dfrac{2v_0}{3}$

$\Rightarrow \left(\dfrac{m}{M}\right) = \dfrac{1}{3} \Rightarrow \left(\dfrac{M}{m}\right) = 3$



Before collision



After collision

5. Object is moving away from the origin till velocity is positive.

6. $\cos\theta = \dfrac{x^2 + R^2 - L^2}{2xR} \Rightarrow 2xR \cos \theta = x^2 + R^2 - L^2$

$2R\left\{-x\sin\theta\dfrac{d\theta}{dt} + \cos\theta\dfrac{dx}{dt}\right\} = 2x\dfrac{dx}{dt}$

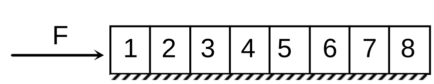
$xv = Rv \cos\theta - Rx\, \omega \sin\theta$

$\Rightarrow v = \left(\dfrac{xR\omega\sin\theta}{R\cos\theta - x}\right)$

7. $F_{81} = ma$

$F_{21} = 7\, ma$

$\dfrac{F_{21}}{F_{87}} = 7$



8. Let Wedge is moving rightward with acceleration a and mass m has an acceleration A with respect to wedge along the surface of the wedge in upward direction, so

$$\frac{h}{\sin\alpha} = \frac{1}{2}At^2 \Rightarrow A = \frac{2h}{t^2 \sin\alpha} \qquad \text{.....(1)}$$

With the help of FBD of mass m in the frame of wedge, we can write

$$A = a\cos\alpha - g\sin\alpha \Rightarrow \frac{2h}{t^2 \sin\alpha} = a\cos\alpha - g\sin\alpha$$

$$\Rightarrow a = g\tan\alpha + \frac{2h}{t^2 \sin\alpha\cos\alpha} = 10\times\frac{3}{4} + 2\times3\times\frac{5}{3}\times\frac{5}{4}\times\frac{1}{5\times5} = 8\text{m}/\text{s}^2$$

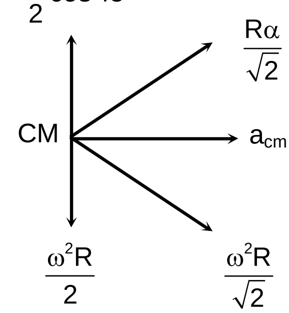
9. Use the concept of graph

10. Using work energy theorem, we get $\Sigma W_1 = \Sigma W_2$
$[W_g + W_{friction}]_1 = [W_g + W_{friction}]_2$
Since, $|f_1| > |f_2| \Rightarrow [W_{ext}]_1 > [W_{ext}]_2$

11. $F - f = Ma$, $fR = \frac{MaR}{2}$
$\Rightarrow$ f = F/3

12. Basic concept of kinematics

13. Net acceleration of c.m. in + y direction = $\frac{R}{2}\cos 45^\circ$
$\frac{R\alpha}{2} - \frac{\omega^2 R}{2}$
$N - mg = ma_{cm,y}$
$N = mg + \frac{m}{2}\left(\alpha - \omega^2\right)R$

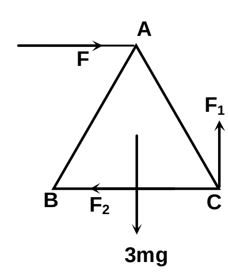


14. F.B.D. of torque

15. Since system is in rotational equilibrium so net torque on the system about C is zero's

$$F_{max} \times \frac{\ell\sqrt{3}}{2} - 3mg\frac{\ell}{2} = 0$$

$$\Rightarrow F_{max} = \sqrt{3}mg$$



16. Using WET, we can write

$$\frac{1}{2}m_c v_c^2 = \frac{-K}{2}\left(x_f^2 - x_i^2\right) - \mu m_0 gd$$

$$\Rightarrow v_c = \sqrt{\frac{kx_i^2 - 2\mu m_c gd}{m_e}}$$

$$v_{sys} = \frac{m_c v_c}{m_e + m_D} = 2\text{ m/s}$$

17. Case I:

$T_1 = 2mg$

$ma_1 = 2mg - T_1$

$\Rightarrow a_1 = g\uparrow$

Case II:

$T_2 = mg$

$3ma_2 = 3mg - T_2$

$\Rightarrow a_2 = 2g/3 \downarrow$

Case III:

$T_3 = 4mg$

$ma_3 = T_3 - mg$

$\Rightarrow a_3 = 3g\uparrow$

Case IV:

$T_4 = mg$

$2ma_4 = 2mg - T_4$

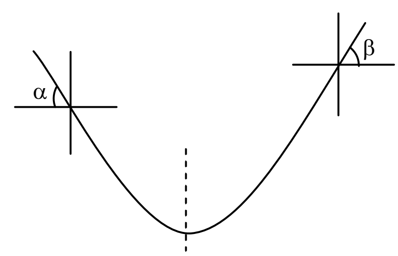
$\Rightarrow a_4 = g/2\downarrow$

18. $T_1\cos\alpha = T_3 = T_2\cos\beta$

$T_1\sin\alpha + T_2\sin\beta = mg$

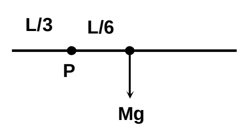
$$\Rightarrow \quad mg = T_1\sqrt{1-\left(\frac{T_3}{T_1}\right)^2} + T_2\sqrt{1-\left(\frac{T_3}{T_2}\right)^2}$$

$$\Rightarrow \quad m = \frac{\sqrt{T_1^2 - T_3^2} + \sqrt{T_2^2 - T_3^2}}{g}$$



19. Use the concept of F.B.D.

20. Torque about P



$$Mg\frac{\ell}{6} = \frac{M\ell^2}{9}\alpha$$

$$\Rightarrow \ \alpha = \frac{3}{2}\frac{g}{\ell}$$

$$\Rightarrow \ a_t = \alpha\frac{\ell}{6} = \frac{g}{4}$$

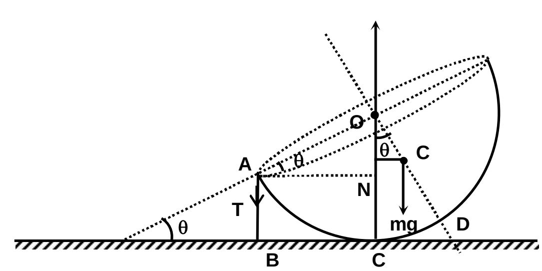
21. According the rotational equilibrium of the hemisphere we can write

$$OC = \frac{3R}{8} \text{ and } ON = R - L = 6 \text{ m}$$

$$AN = \sqrt{R^2 - (R-L)^2} = 8 \text{ m}$$

$$T \times R\cos\theta - mg\frac{3R}{8}\sin\theta = 0$$

$$T = \frac{3mg\tan\theta}{8} = \frac{3\times 64\times 10\times 6}{8\times 8} = 180 \text{ N}$$



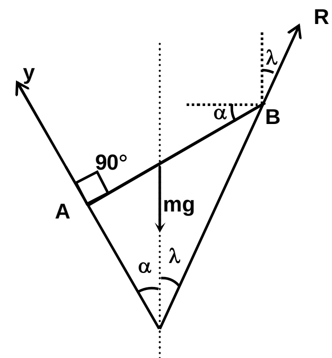
22. $2T - Mg + T = Ma \Rightarrow Mg - 3T = Ma \Rightarrow 3T = M(g - a)$ ….(i)

$2T = kx$

$$T = \frac{M(g+a)}{3}$$

Reading of spring balance = $\frac{kx}{g} = \frac{2T}{g} = \begin{cases} \frac{2M(g+a)}{3g} & \text{(when moving upward)} \\ \frac{2M(g-a)}{3g} & \text{(when moving downward)} \end{cases}$

23. Since body is in equilibrium, under the influence of three forces only so they must be concurrent. Using Lami's theorem we can write



$\tan\alpha = \frac{L}{2x}$ and $\tan(\alpha+\lambda) = \frac{L}{x}$

$\tan(\alpha + \lambda) = 2\tan\alpha \Rightarrow \frac{\tan\alpha + \tan\lambda}{1-\tan\alpha\tan\lambda} = 2\tan\alpha$

$\Rightarrow \tan\alpha + \tan\lambda = 2\tan\alpha - 2\tan^2\alpha\tan\lambda$

$\Rightarrow \tan\lambda = \mu = \frac{\tan\alpha}{1+2\tan^2\alpha}$

24. $v = \begin{cases} \frac{2t^2}{9} & \text{if } 0 \le t \le 15 \text{ sec} \\ 150 + 30(t-15) & \text{if } 15 \text{ s} \le t \le 40 \text{ s} \end{cases}$

at t = 15 sec

$v = \frac{3\times 15\times 15}{3} = 150$ m/s

$\vec{r} = \begin{cases} \frac{2t^3}{3}\hat{j} & \text{if } 0 \le t \le 15 \text{ sec} \\ 150 + 150(t-15) + 15(t-15)^2\hat{j} & \text{if } 15 \text{ s} \le t \le 40 \text{ s} \end{cases}$

at t = 15

$r = \frac{2\times 15\times 15\times 15}{3\times 3} = 750$ m

25. $v_x = 1 \Rightarrow x = t$ and $v_y = 6t \Rightarrow y = 3t^2 \Rightarrow y = 3x^2$

$\Rightarrow \frac{dy}{dx} = 6x, \ \frac{d^2y}{dx^2} = 6 \Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{2}}{3}} = 2\sqrt{2}$

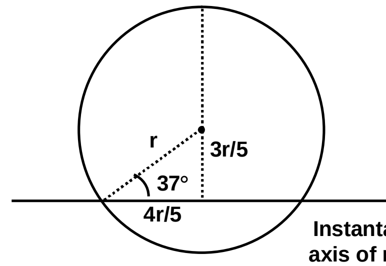
As we know that

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(1+8)^{3/2}}{6} = 4.5 \text{ m}$$

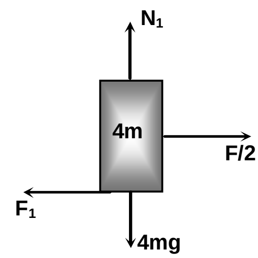
26. $8x_B = x_A \Rightarrow 8v_B = v_A \Rightarrow 8a_B = a_A$

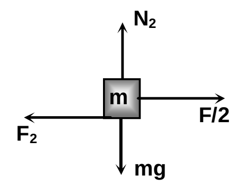
27. $v = \omega \times \frac{3r}{5} \Rightarrow \omega = \frac{5v}{3r}$

$v_B = \omega \times \frac{8r}{5} = \frac{5v}{3r} \times \frac{8r}{5} = \frac{8v}{3}$



28. Both blocks 4m and m has tendency of motion towards rightwards, so friction forces (F_1 and F_2) will act on both blocks leftward, hence on the platform rightward, so





$a = \frac{F_1 + F_2}{m} = 2\,m/s$

$F_1 + F_2 = 20$ N

$F_1 \le 64\ N_1\ F_2 \le 16$ N

Case I: Suppose block of mass 4m has a relative motion with respect to platform

So, $F_{1k} = 40$ N

$F_2 = -20$ N (not possible)

Case II: Suppose block of mass m has a relative motion with respect to platform, so

$F_{2k} = 10\ N \Rightarrow F_{1s} = 10$ N (possible)

$\frac{F}{2} - F_{1s} = 4m \times 2 \Rightarrow \frac{F}{2} = 1) + 4 \times 10 \times 2 = 90$

$\Rightarrow F = 180\ N \Rightarrow \frac{F}{2} - F_{2k} = ma \Rightarrow 90 - 10 = 10a \Rightarrow a = 8\ m/s^2$

29. First Method:

$$I_{AB} = \int y^2 dm = \frac{\lambda_0}{\ell^3} \int_{-x}^{\ell - x} (y+x)^3 y^2 dy$$

$$\int (y+x)^3 y^2 dy = y^2 \frac{(y+x)^4}{4} - \frac{1}{2} y(y+x)^4 dy$$

$$= \frac{y^2 (y+x)^2}{4} - \frac{y}{10}(y+x)^5 + \frac{1}{10}(y+x)^5 dy$$

$$= \frac{y^2 (y+x)^2}{4} - \frac{y(x+y)^5}{10} + \frac{(y+x)^2}{60}$$

$$\int_{-x_0}^{(\ell - x)} y^2 (y+x)^3 dx = \frac{(L-x)^2 (\ell)^4}{4} - \frac{(\ell - x)\ell^5}{10} + \frac{\ell^6}{60}$$

$$I_{AB} = \lambda_0 \left[\frac{(\ell - x)^2}{4} \ell - \frac{1}{10}(\ell - x)\ell^2 + \frac{1}{60}\ell^3 \right] = \lambda_0 \ell \left[\frac{(\ell - x)^2}{4} \ell - \frac{1}{10}(\ell - x) + \frac{1}{60}\ell^2 \right]$$

$$\frac{dI_{AB}}{dx} = 0 \Rightarrow -\frac{(\ell - x)}{2} + \frac{\ell}{10} = 0 \Rightarrow 5\ell - 5x = \ell \Rightarrow 5x = 4\ell \Rightarrow x = \frac{4\ell}{5}$$

Second Method:

As we know that moment of inertia is minimum among all set of parallel axis, if axis is passing through centre of mass, so x must be distance of centre of mass.

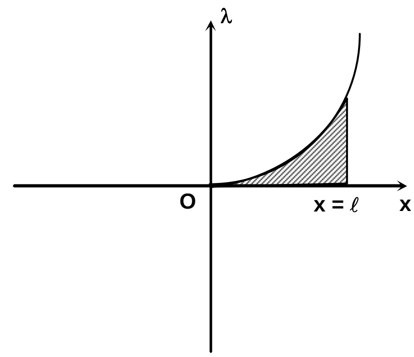
$$x_{CM} = \frac{1}{M}\int_0^L x\,dm \qquad \ldots.(i)$$

$$M = \int dm = \frac{\lambda_0}{\ell^3}\int_0^{\ell} x^3dx = \frac{\lambda_0\ell}{4}, \text{ and } \int_0^{\ell} x\,dx = \frac{\lambda_0}{\ell^3}\int_0^{\ell} x^4dx = \frac{\lambda_0\ell^2}{5}$$

Putting these values in equation (i) we have $x_{cm} = \frac{4\ell}{5}$

Third Method:

From given function it is clear that centre of mass of system must be at distance grater than $\ell/2$. We have only two option, $\frac{8\ell}{15}$ and $\frac{4\ell}{5}$. For graph it is clear that maximum mass existence is nearer to other end, so it can't be $\frac{8\ell}{15}$, so right answer will be $\frac{4\ell}{5}$.



30. Time taken by Car-A to reach the finish line is greater than that of Car-B, So

$$\int_0^{t_A} F\,dt > \int_0^{t_B} F\,dt \Rightarrow \Delta p_A > \Delta p_B$$

Chemistry

PART – II

SECTION – A

31. $\psi(3s) = 0$

$6 - 6\sigma + \sigma^2 = 0$

$\sigma = -\dfrac{(-6) \pm \sqrt{36-(4(1)6)}}{2\times 1}$

$\sigma = \dfrac{6 \pm \sqrt{12}}{2}$

$\sigma = \dfrac{6 \pm 2\sqrt{3}}{2}$

$\sigma = 3+\sqrt{3}$ or $3-\sqrt{3}$

$\dfrac{2rZ}{3a_o} = 3+\sqrt{3}$

$r = \dfrac{3}{2}\left(3+\sqrt{3}\right)\dfrac{a_o}{Z}$

32. $[Cr] = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^5$

For p $\ell = 1$

m = +1, 0, -1

For 2p of 3p there are 2e with $s = -\dfrac{1}{2}$

For d $\ell = 2$

m = +1 may have e with $+\dfrac{1}{2}$ or $-\dfrac{1}{2}$

So minimum 2 and maximum 3.

33.

$\uparrow\uparrow$	$\uparrow\downarrow$	

Same value of spin quantum number.

34. K_{eq} at 1400 K $= \dfrac{0.29}{1.1\times 10^{-6}} = 0.26\times 10^6$

$= 26\times 10^4$

K_{eq} at 1500 K $= \dfrac{1.3}{1.4\times 10^{-5}} = 0.92\times 10^5$

$= 92\times 10^3$

Since K_{eq} decreases on increasing temperature. So reaction is exothermic.

35.

	$A \longrightarrow$	$2B$	$+\ C$
$t = 0$	P_A		
$t = t$	$P_A - P'$	$2P'$	P'
$t = \infty$	0	$2P_A$	P_A

$P_A - P' + 2P' + P' = P_t$

$P_A + 2P' = P_t$
$2P_A + P_A = P_\infty$
$P_A = \frac{P_\infty}{3}$
$\frac{P_\infty}{3} + 2P' = P_t$
$P' = \left(P_t - \frac{P_\infty}{3}\right)\frac{1}{2}$
$Kt = \ell n\frac{P_A}{P_A - P'} = \ell n\frac{P_\infty}{3\left(\frac{P_\infty}{3} - \frac{P_t}{2} + \frac{P_\infty}{6}\right)}$
$Kt = \ell n\frac{P_\infty}{3\left(\frac{2P_\infty - 3P_t + P_\infty}{6}\right)}$
$Kt = \ell n\frac{2P_\infty}{3P_\infty - 3P_t}$
$K = \frac{2.303}{t}\log\frac{2(P_\infty)}{3(P_\infty - P_t)}$

36. Zero order reaction is always a complex reaction.

37. $\log\frac{K_2}{K_1} = \frac{\Delta H}{R \times 2.303}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$
If $\Delta H = 0$
Then $K_2 = K_1$
Means no effect.

38. For exothermic reaction high temperature favour backward reaction and with increase in pressure reaction goes where number of moles are less.

39. $2A(g) \rightleftharpoons 2B(g) + C(g)$
v
$v - 2v'$ $\quad 2v' \quad v'$
$v' = 100$
$v - 2v' + 2v' + v' = 700$
$v = 600$
$2A(g) \rightleftharpoons 2B(g) + C(g)$
400 $\quad$ 200 $\quad$ 100
$10 \times \frac{400}{700}$ $\quad 10 \times \frac{200}{700}$ $\quad 10 \times \frac{100}{700}$
$= \frac{40}{7}$ $\quad \frac{20}{7}$ $\quad \frac{10}{7}$
$K_P = \frac{\left(\frac{20}{7}\right)^2 \times \frac{10}{7}}{\left(\frac{40}{7}\right)^2} = \frac{20 \times 20 \times 10}{40 \times 40 \times 7} = \frac{10}{28}$

40. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ is 4.5

P

$P - P\alpha \qquad 2P\alpha$

$P - P\alpha + 2P\alpha = 2$

$P + P\alpha = 2$

$$4.5 = \frac{4P^2\alpha^2}{P(1-\alpha)}$$

$$4.5 = \frac{4P\alpha^2}{1-\alpha}$$

$4.5(1-\alpha) = 4P\alpha^2$

$$P = \frac{4.5(1-\alpha)}{4\alpha^2}$$

$P(1+\alpha) = 2$

$$\frac{4.5(1-\alpha)(1+\alpha)}{4\alpha^2} = 2$$

$4.5(1-\alpha^2) = 8\alpha^2$

$4.5 - 4.5\alpha^2 = 8\alpha^2$

$4.5 = 12.5\alpha^2$

$$\alpha = \sqrt{\frac{4.5}{12.5}}$$

$\alpha = 0.6$

$$\alpha = \frac{M - EMM}{EMM(n-1)}$$

$$0.6 = \frac{92 - EMM}{EMM(2-1)}$$

$0.6EMM = 92 - EMM$

$1.6EMM = 92$

$$EMM = \frac{92}{1.6}$$

$= 57.5$

41. $HA \rightleftharpoons H^+ + A^-$

0.25

$0.25 - x \qquad x \qquad x$

$\simeq 0.25$

$$4 \times 10^{-8} = \frac{x^2}{0.25}$$

$$x^2 = 4 \times \frac{25}{100} \times 10^{-8}$$

$x = 10^{-4}$

$x = \left[H^+\right] = 10^{-4}$

42. Due to common ion effect solubility decreases.

43. When AgCl start precipitating concentration of $[Ag^+]$ in the solution

$$\left[Ag^+\right] = \frac{K_{spAgCl}}{\left[Cl^-\right]} = \frac{1.8\times10^{-10}\times10}{0.1}$$

$$\left[Ag^+\right] = 1.8\times10^{-9}$$

$$K_{spAgI} = \left[Ag^+\right]\left[I^-\right]$$

$$1.5\times10^{-16} = 1.8\times10^{-9}\left[I^-\right]$$

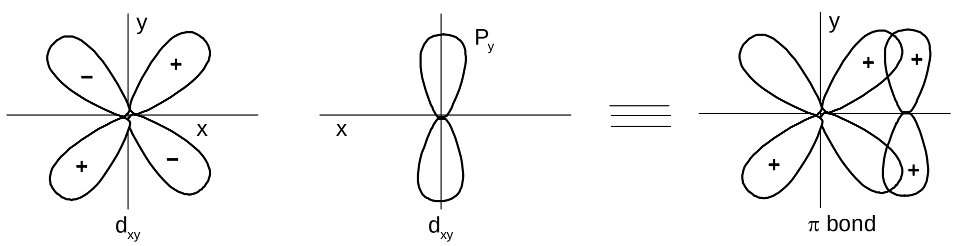
$$\left[I^-\right] = \frac{1.5\times10^{-16}}{1.8\times10^{-9}} = 0.833\times10^{-7}$$

$$= 8.3\times10^{-8}$$

44. More is the concentration of common ion, lesser is the solubility.

45. Bond angle in XeF_4 and $XeCl_4$ is equal.

46. Fact

47. 

48. sp^3d^2 orbital involved are d_{z^2} and $d_{x^2-y^2}$

49. Smaller is the size, larger is the hydration energy.

50. More is covalent character, lesser is the thermal stability.

51. More is the ionic character, stronger is the base.

52. Conceptual.

53. Na^+ have maximum ionization energy because of noble gas configuration and Na have lowest ionization energy because electron have to remove from 3rd shell.

54. More is the negative charge, larger is the size.

55. They can accept pair of electron because of vacant orbital.

56. Fact.

57. Because of layer of structure.

58. $5H_2O_2 + 2ClO_2 + 2OH^- \longrightarrow 2Cl^- + 5O_2 + 6H_2O$

59. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

60. Larger is the size of cation more is the ionic character.

Mathematics

PART – III

SECTION – A

61. Use substitution

$x - 1 \to X$

$y + 2 \to Y$

To make it homogenous differential equation

$$\frac{dY}{dX} = \frac{(X+Y)^2}{XY}$$

$\Rightarrow Y = vX$

$$\Rightarrow X\frac{dv}{dX} = \frac{1+2v}{v}$$

$$\Rightarrow \frac{1}{2}v - \frac{1}{2\cdot 2}\ln|1+2v| = \ln|X| + c$$

62. $x = 0,\ y = \frac{x}{2}$

$\Rightarrow f(x) + f(0) = 2f(x) + \frac{x^2}{2}$

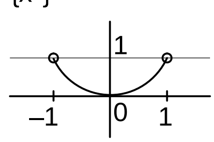
$\Rightarrow f(0) - \frac{x^2}{2} = f(x)$

$\Rightarrow$ f(x) is into, many-one, non-invertible

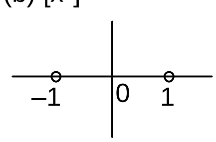
63. $$\lim_{n\to\infty}\left[1+\frac{\sum r(n-r+1)}{n^2(n+1)^2/4}\right]^n = e^{\frac{n\left[n^2(n+1)/2 - n(n+1)(2n+1)/6 + n(n+1)/2\right]}{n^2(n+1)^2/4}} = e^{\frac{1/2-1/3}{1/4}} = e^{\frac{2}{3}}$$

64. Shown in the figures

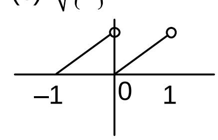
(a) $\{x^2\}$



(b) $[x^2]$



(c) $\sqrt{\{x\}^2}$



65. $$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right) = \frac{dx}{dy}\frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1} = \frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

$$\frac{d^3x}{dy^3} = \frac{d}{dy}\left(\frac{d^2x}{dy^2}\right) = \frac{dx}{dy}\frac{d}{dx}\left(-\frac{d^2y}{dx^2}.\left(\frac{dy}{dx}\right)^{-3}\right) = \frac{-d^3y}{dx^3}\left(\frac{dy}{dx}\right)^{-4} + 3\left(\frac{dy}{dx}\right)^{-5}\left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow \left(\frac{d^3x}{dy^3}\right)\left(\frac{dy}{dx}\right)^5 = 3\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx}\frac{d^3y}{dx^3}$$

66. $$\int\left(x^9 + x^4 - x^4\right)\left(1+x^5\right)^{2/5}dx = \int x^4\left(x^5+1\right)^{7/5}dx - \int x^4\left(1+x^5\right)^{2/5}dx$$

$$= \frac{(1+x^5)^{12/5}}{12} - \frac{(1+x^5)^{7/5}}{7} + C$$

67. $\left[\frac{3f(x)-|f(x)|}{3f(x)+|f(x)|}\right] = \begin{cases} 0 & \text{if} \quad f(x) \ge 0 \\ 2 & \text{if} \quad f(x) < 0 \end{cases}$

$\Rightarrow \int_0^1 2dx + \int_2^3 2dx + \int_4^5 2dx = 6$

68. $f(x+\pi) = \cos(\pi \sin^2(x+\pi)) = f(x)$

$g(x+\pi) = \cos(\pi \cos^2(x+\pi)) = g(x)$

$h(x) = f(x) + g(x) = 2\cos\left(\frac{\pi}{2}\right).\cos(\pi\cos(2(x+\pi))) = 0$

h(x) periodic without any fundamental period

69. $\tan^{-1}x$ is increasing function

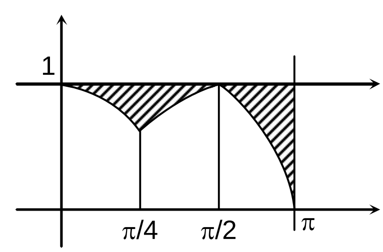
$\Rightarrow T_n < S_n$

$\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} T_n = \int_0^1 \tan^{-1} x dx = \frac{\pi - \ln 4}{4}$

$S_n > \frac{\pi - \ln 4}{4}, T_n < \frac{\pi - \ln 4}{4}$

70. Third root will have to be zero

71. $\frac{\sin x + \cos x + |\sin x - \cos x|}{2} = \begin{cases} \cos x & x \in \left[0, \frac{\pi}{4}\right] \\ \sin x & x \in \left[\frac{\pi}{4}, \pi\right] \end{cases}$



Required Area = $\int_0^{\pi/4} (1-\cos x)dx + \int_{\pi/4}^{\pi/2} (1-\sin x)dx + \frac{\pi}{2} - 1$

$= \pi - \sqrt{2} - 1$

72. $\text{Sgn}\left(x^2 - \frac{\pi x}{2}\right)$ is discontinuous at x = 0 and x = $\frac{\pi}{2}$ and (1 – sin x – cos x) $\to$ 0

As x $\to$ 0 and x $\to \frac{\pi}{2}$. So f(x) is continuous $\forall$ x.

73. Domain of f(x) = $(-\infty, 0)$

Range of g(x) = $[-1, 1]$

$\Rightarrow$ Domain of (fog) (x) = $(-1, 0)$

74. $\left|\frac{y}{m}\right|^2 \propto |ym| \Rightarrow \frac{y^2}{m^2} = kym$

$\Rightarrow$ y = 0 or $\frac{y}{k} = m^3$

$\Rightarrow \dfrac{dy}{y^{1/3}} = \dfrac{dx}{k^{1/3}} \Rightarrow \dfrac{3}{2}y^{2/3} = \dfrac{x}{k^{1/3}} + C$

$\Rightarrow y^2 = \dfrac{2}{3k^{1/3}}\left(x + Ck^{1/3}\right)^3$

If $\dfrac{2}{3k^{1/3}}$ = 1 and $Ck^{1/3} = -2$

$\Rightarrow y^2 = (x-2)^3$

75. Let $C_1(x) = e^{100x}f(x)$

$\Rightarrow C_1'(x) = e^{100x}g(x)$

If $C'_{(x)} = 0 \Rightarrow g(x) = 0$

If f(x) has roots $\alpha_1, \alpha_2, \ldots, \alpha_n$ then applying Rolle's theorem in (α_1, α_2), (α_2, α_3) ..... (α_{n-1}, α_n), we get g(x) has atleast n – 1 roots

76. Let $f(x) = x^{c/x}$ where c is a constant then f(x) has maxima at x = e

(a) use function $f(x) = x^{\frac{e\pi}{x}} \Rightarrow f(e) > f(\pi) \Rightarrow e^{\pi} > \pi^e$

(b) $f(x) = x^{\frac{3\pi}{x}}$ $f(3) > f(\pi) \Rightarrow 3^{\pi} > \pi^3$

(c) $f(x) = x^{\frac{2e}{x}} \Rightarrow f(e) > f(2) \Rightarrow e^2 > 2^e$

(d) $f(x) = x^{\frac{10\pi}{x}} \Rightarrow f(\pi) > f(10) \Rightarrow \pi^{10} > 10^{\pi}$

77. $\int_{-2}^{2} x^4 d(\ln x)$

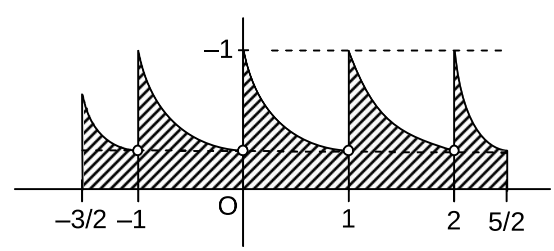
Let ln x = t $\Rightarrow$ d (ln x) = dt

$\int_{-2}^{2} e^{4t}dt \Rightarrow \left|\dfrac{e^{4t}}{4}\right|_{-2}^{2} = \dfrac{e^8 - e^{-8}}{4}$

78. $f(x) = \dfrac{1}{(\{x\}+1)^2}$

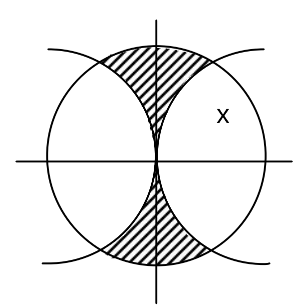
$\int_{-3/2}^{5/2} f(x)dx = 4\int_0^1 \dfrac{1}{(\{x\}+1)^2}\,dx$

$= 4\int_0^1 \dfrac{1}{(1+x)^2}\,dx = 4\left|\dfrac{-1}{1+x}\right|_0^1 = 2$



79. Required area can be through of as 4[area of quadrant of circle – x]

Where $x = \int_0^{\sqrt{20}} \left(\sqrt{36-y^2} - \dfrac{y^2}{5}\right) dy$



80. $\displaystyle\int \frac{6x^{23}-9x^{8}}{\left(x^{15}+x^{9}+1\right)^{2}}\,dx = \int \frac{6x^{5}-9x^{-10}}{\left(x^{6}+1+x^{-9}\right)^{2}}\,dx$

$\displaystyle = \frac{-1}{x^{6}+x^{-9}+1}+C = \frac{-x^{9}}{x^{15}+x^{9}+1}+C$

81. $\displaystyle\lim_{x\to\infty}\frac{\sin x}{x}=0$

$\displaystyle\frac{\lim\limits_{x\to\infty}\int_0^x e^t dt = 0}{e^{x^2}} = 0$

$\displaystyle\lim_{x\to\infty}\left(\sin x\right)^{1/x^2}=0$

82. (p ∨ ~ r) ∨ (~p ∧ q ∧ r) = (p ∨ ~ r) ∨ (~(p ∨ ~ r) ∧ q) = (p ∨ ~ r) ∨ q = ~(~p ∧ ~ q ∧ r)

83. $\displaystyle\int \frac{\left(3x^{2}+1\right)dx}{\left(x^{3}+x+1\right)^{2}+1} = \tan^{-1}\left(x^{3}+x+1\right)+C$ $= -\cot^{-1}(x^3 + x - 1) + C$

84. $\sqrt{2y+x}+\sqrt{2y-x}=C$

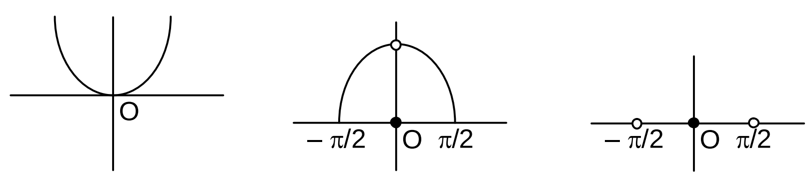
$\displaystyle\Rightarrow \frac{2y'+1}{2\sqrt{2y+x}}+\frac{2y'-1}{\sqrt{2y-x}}=0$

Rearrange and rationalize to get

$\displaystyle y' = \frac{x}{4y+2\sqrt{4y^{2}-x^{2}}}$

85. Shown in the figure

$f(x) = |x|^5$ $f(x) = \{\cos x\}$ $h(x) = [|\sin x|]$

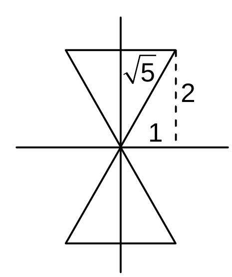


86. $V = 2\ \frac{\pi}{3}\ r^2h$

$= 2.\ \frac{\pi}{3}.\ 1.\ 2 = \frac{4\pi}{3}$

$S = 2\ \pi rl$

$= 2\sqrt{5}\ \pi$



87. **Reflexive** ∀ f ∈ R, f – f = 0 ∈ R
⇒ (f, f) ∈ R
Symmetric ∀ (f, g) ∈ R
⇒ f – g is even
⇒ g – f is even
⇒ (g, f) ∈ R
Transitive ∀ (f, g) ∈ R, (g, h) ∈ R

$\Rightarrow$ f – g is even, g – h is even
$\Rightarrow$ f – h is even
$\Rightarrow$ (f, h) $\in$ R
So, R is equivalence relation

88. For area to be minimum $y = e^{1/8} = e^{a(1/2)^2}$

$\Rightarrow a = \frac{1}{2}$

89. sin(y′) + 3 cosy′ = 2xy

$\Rightarrow \sqrt{10}\sin(y' + \phi) = 2xy$

$\Rightarrow y' = \sin^{-1}\frac{2xy}{\sqrt{10}} - \phi$

90. Number of possible pairs of (x, y) = ${}^nC_0 2^n + {}^nC_1 2^{n-1} + ... + {}^nC_n 2^{n-n}$