

PART-I : PHYSICS

SECTION-I (Single Correct Choice Type)

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

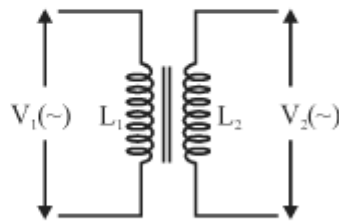
1. For an ideal transformer, ratio of V_1 & V_2 is equal to
Where L_1 & L_2 are self inductances of primary and secondary windings. (For an ideal transformer coefficient of mutual induction for pair of coils is $M = \sqrt{L_1 L_2}$):

(A) $\frac{L_1}{L_2}$

(B) $\sqrt{\frac{L_1}{L_2}}$

(C) $\frac{L_2}{L_1}$

(D) $\sqrt{\frac{L_2}{L_1}}$



2. According to Sommerfield model of atom an electron revolve around nucleus in an elliptical orbit while nucleus remain at rest. If minimum and maximum distance of electron from nucleus is r_1 & r_2 then find distance of electron from nucleus when it is at a position where line joining electron & nucleus is perpendicular to major axis :-

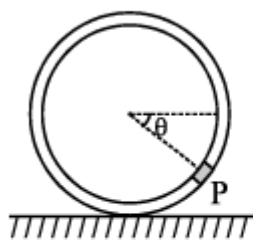
(A) $r_1 + r_2$

(B) $r_2 - r_1$

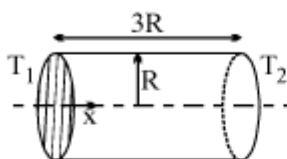
(C) $\frac{r_1 r_2}{r_1 + r_2}$

(D) $\frac{2r_1 r_2}{r_1 + r_2}$

3. A small block of mass m is rigidly attached at 'P' to a ring of mass m and radius r . The system is released from rest at $\theta = 30^\circ$ and rolls without sliding. The speed of centre of mass of system when block reaches the bottom is



- (A) \sqrt{rg}
 (B) $\frac{\sqrt{rg}}{2}$
 (C) $\sqrt{\frac{rg}{2}}$
 (D) none
4. One mole of an ideal monoatomic gas is enclosed in a chamber at 300 K. The gas undergoes a process in which pressure is proportional to the volume. At the end of the process, the volume of the gas is doubled. The change in the internal energy of the gas is [R is gas constant]
 (A) 450 R
 (B) 700 R
 (C) 1350 R
 (D) data insufficient
5. For a conducting cylinder with variable thermal conductivity along the axis the magnitude of rate of heat flow if temperature difference across ends is $(T_1 - T_2)$,



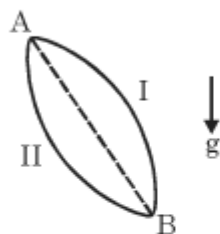
solid cylinder, flow along axis,
 variable k as $k = k_0(1 + x/(3R))$
 x is distance from left end

- (A) $6\pi k_0 R(T_1 - T_2)$
 (B) $\frac{\pi k_0 R}{3 \ln 2}(T_1 - T_2)$
 (C) $\pi k_0 R(T_1 - T_2)$
 (D) $\frac{4\pi k_0 R}{\ln 2}(T_1 - T_2)$

SECTION-II (More than one Correct Choice Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

6. A small bob can slide downwards from point A to B along either of the two different curved surfaces shown in cross-section in the diagram. These possible trajectories are circular arcs in vertical plane, and they lie symmetrically about straight line AB. During the motion, bob does not leave contact with surface.



- (A) If friction is neglected every where bob will take a smaller time through path I as compared to time through path II.
 (B) If friction is neglected every where bob will take a smaller time through path II as compared to time through path I.
 (C) If friction is neglected every where bob will reach point B with same speed through both the paths.
 (D) If friction is significant along both paths then bob will reach B with a smaller speed, if it follows path II, as compared to if it follows path I.
7. On a spherical planet, the refractive index of atmosphere, as a function of altitude h above the surface varies as

$$n(h) = \frac{n_0}{1 + \frac{h}{2R}}$$

Where n_0 is a constant. A laser beam directed horizontally but an arbitrary altitude follows a trajectory that circles the planet.

- (A) Radius of planet has a magnitude $R\sqrt{2}$
 (B) Radius of planet has a magnitude $2R$
 (C) At a height $R\sqrt{2}$ from the surface of planet, refractive index is $\frac{n_0}{2}$
 (D) At a height R from surface, refractive index is $\frac{2n_0}{3}$

8. Two equal masses m are at a distance ℓ apart and interact via gravity. They are given proper tangential speed v_0 , so that they both travel in a circular of radius $\frac{\ell}{2}$ around their CM. If one of the masses is grabbed and held at rest, then the closest distance the other mass comes to it is r . Then :

(A) $v_0 = \sqrt{\frac{Gm}{2\ell}}$

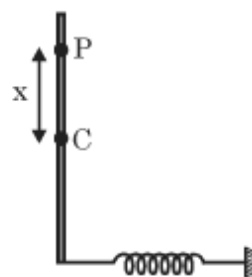
(B) $v_0 = \sqrt{\frac{Gm}{\ell}}$

(C) $r = \frac{\ell}{3}$

(D) $r = 3\ell$

9. A uniform stick with mass m and length ℓ lies on a frictionless horizontal surface. It is pivoted at a point a distance x from centre. A spring (at its relaxed length) with spring constant k is attached to the far end of the stick, perpendicular to the stick, as shown.

Stick is given a small kick, then :



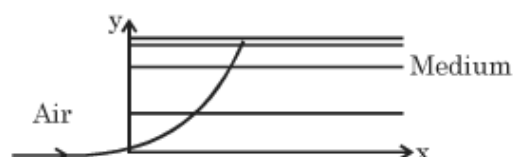
(A) Value of x which yields largest frequency of oscillation is $\frac{\ell}{3}$.

(B) Value of x which yields largest frequency of oscillation is $\frac{\ell}{6}$.

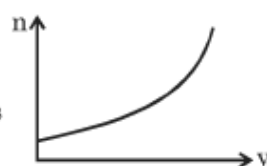
(C) Angular frequency of oscillation (largest value) is $\frac{2k}{m}$.

(D) Angular frequency of oscillation (largest value) is $\frac{4k}{m}$.

10. The refractive index of the medium within a certain region $x > 0, y > 0$, changes continuously with y . A thin light ray travelling in air in the x -direction strikes the medium at right angles and moves through the medium along a circular arc of radius R .



- (A) Refractive index of medium varies with y as



- (B) Refractive index of medium varies with y as

- (C) If refractive index of medium can increase upto a value $n = 2.5$, the maximum

value of y is $\frac{3R}{5}$

- (D) If refractive index of medium can increase upto a value $n = 2.5$, the maximum value of y is $5R$

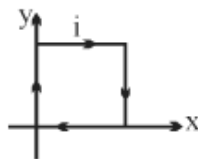
11. Two waves given by equation $y_1 = A \sin(\omega t - kx)$ and $y_2 = 4A \sin(2\omega t - 4kx)$ are traveling in two different strings having same mass per unit length and same cross-sectional area. Tension in first string is four times tension in the second string.

- (A) Ratio of average energy density U_1/U_2 is $1/64$
 (B) Ratio of average intensity I_1/I_2 is $1/32$
 (C) Ratio of average power P_1/P_2 is $1/16$
 (D) Ratio of velocity of wave v_1/v_2 is $1/2$

12. A metal block is placed in a room which is at 10°C for long time. Now it is heated by an electric heater of power 500 W till its temperature becomes 50°C . Its initial rate of rise of temperature is 2.5°C/sec . The heater is switched off and now a heater of 100 W is required to maintain the temperature of the block at 50°C . (Assume Newtons Law of cooling to be valid)

- (A) The heat capacity of the block is $200\text{ J/}^\circ\text{C}$
 (B) The heat capacity of the block is $100\text{ J/}^\circ\text{C}$
 (C) The rate of cooling of block at 50°C if the 100 W heater is also switched off is 0.5°C/s
 (D) The heat radiated per second when the block was 30°C is 50 W

13. Figure shows a square current carrying coil of edge length L . The magnetic field on the coil is given by $\vec{B} = \frac{B_0 y}{L} \hat{i} + \frac{B_0 x}{L} \hat{j}$ where B_0 is a positive constant. (A is area of coil)

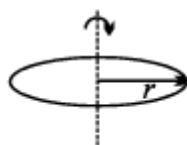


- (A) If coil is free to rotate about x axis torque on the coil is given by $\frac{1}{2} i A B_0 \hat{i}$.
- (B) If coil is free to rotate about y - axis torque on coil is given by $-\frac{1}{2} i A B_0 \hat{j}$.
- (C) Resultant force on coil is zero.
- (D) Equation for the torque $\vec{\mu} \times \vec{B}$ where μ is magnetic moment of coil is not valid on the coil if any of the side is fixed as axis.

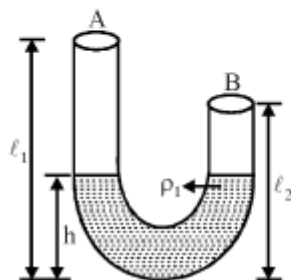
SECTION-III (Integer Type)

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14. A ring of radius r made of wire of density ρ is rotated about a stationary vertical axis passing through its centre and perpendicular to the plane of the ring as shown in figure. Find the percentage change in tension for 3 percent change in angular velocity of rotation. Ignore gravity.

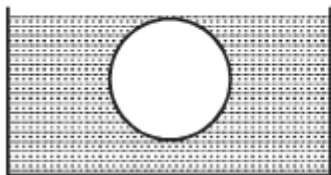


15. A U-tube having uniform cross-section but unequal arm length $l_1 = 100$ cm and $l_2 = 50$ cm has same liquid of density ρ_1 filled in it upto a height $h = 30$ cm as shown in figure. Another liquid of density $\rho_2 = 2\rho_1$ is poured in arm A. Both liquids are immiscible. What length of the second liquid (in cm) should be poured in A so that second overtone of A is in unison with fundamental tone of B. (Neglect end correction)



16. A source of frequency 175 Hz is placed between a man and a wall at the same height of person's ear. Find the velocity v (assume $v \ll c$) of the source with which it should approach the wall such that person will detect 3 beats per second. [Take velocity of sound $c = 350$ m/s]

17. A sphere is floating in a liquid in just submerged position as shown. Find the ratio of magnitude of forces exerted by liquid on lower half of sphere to that exerted by liquid on upper half of sphere (Neglect atmospheric pressure)



18. One mole of an ideal monoatomic gas is taken from state A to state B through the process $P = \frac{3}{2}T^{1/2}$. It is found that its temperature increases by 100 K in this process. Now it is taken from state B to C through a process for which internal energy is related to volume as $U = \frac{1}{2}V^{1/2}$. The total work performed by the gas (in Joule) is W, if it is given that volume at B is 100 m^3 and at C it is 1600 m^3 . [Use $R = 8.3 \text{ J/mol-K}$]. Fill W/87.

PART-II : CHEMISTRY

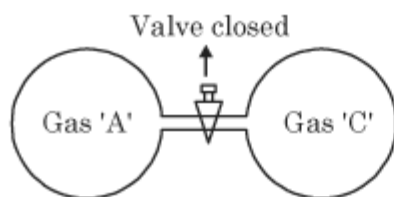
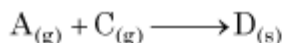
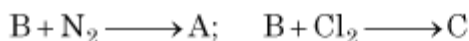
SECTION-I (Single Correct Choice Type)

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19. Which one of the following dimeric form of compounds is planar ?

(A) B_2H_6
(B) Al_2Cl_6
(C) Al_2Br_6
(D) I_2Cl_6

20. Borazine + $H_2O \longrightarrow H_3BO_3 + A_{(g)} + B_{(g)}$



Flask - I

$T = 300\text{ K}$

$P = 485\text{ mm Hg}$

Flask - II

$T = 400\text{ K}$

$P = 600\text{ mm Hg}$

Two flasks – I & II of equal volume containing Gas 'A' and Gas 'C', are connected by a narrow tube of negligible volume. The two gases were prevented from mixing by stopper fitted in connecting tube. For further detail of experiment refer to the given figure. What will be final pressure in each flask when valve connecting two tubes is opened. Assume ideal gas behaviour.

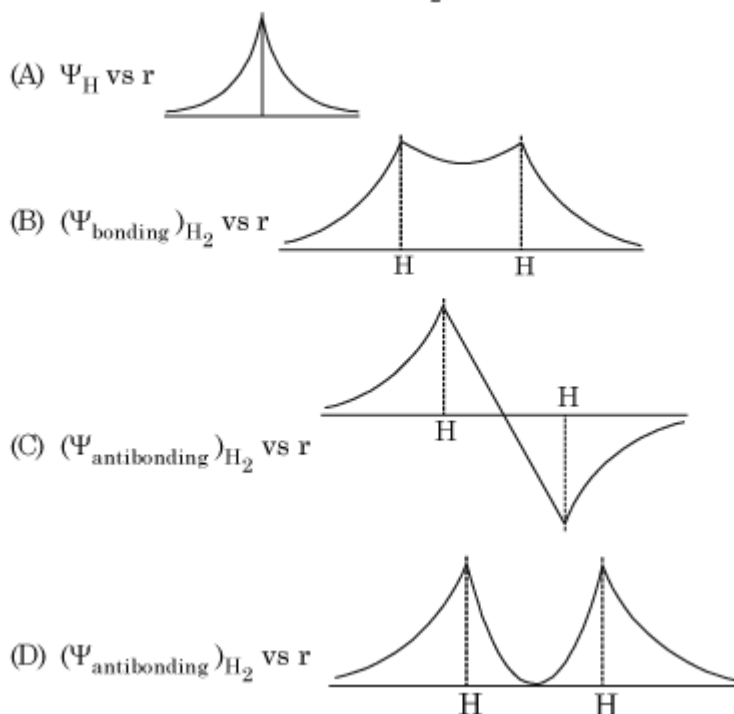
(A) 40 mm Hg
(B) 60 mm Hg
(C) 20 mm Hg
(D) 10 mm Hg

21. A single walled carbon nano-tube can be regarded as a single sheet of graphite rolled into a cylinder. Number of C-atom in a single walled nano-tube having length 1.0 mm & diameter 1.08 nm ?

[Given : Carbon - Carbon bond length in graphite = 141.5 pm]

(A) 0.65×10^8
(B) 2.6×10^8
(C) 3.9×10^8
(D) 1.3×10^8

22. For the interaction $H + H \longrightarrow H_2$, **INCORRECT** is :



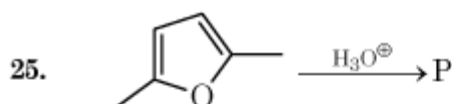
23. Identify the **CORRECT** option.

- (I) AgCl is not soluble in H_2O but soluble in KCN .
 (II) Zn when reacts with very dil HNO_3 (6%) gives NH_4NO_3 .
 (III) CuSO_4 reacts with KI to form only $\text{Cu}_2\text{I}_2 + \text{K}_2\text{SO}_4$.
 (IV) $\text{CuSO}_4 \cdot 5\text{H}_2\text{O} \xrightarrow{1000^\circ\text{C}}$ The gases liberated are SO_2 and O_2 .
- (A) All statements are correct
 (B) I, II and IV are correct
 (C) II and III are incorrect statements
 (D) Only I is incorrect.

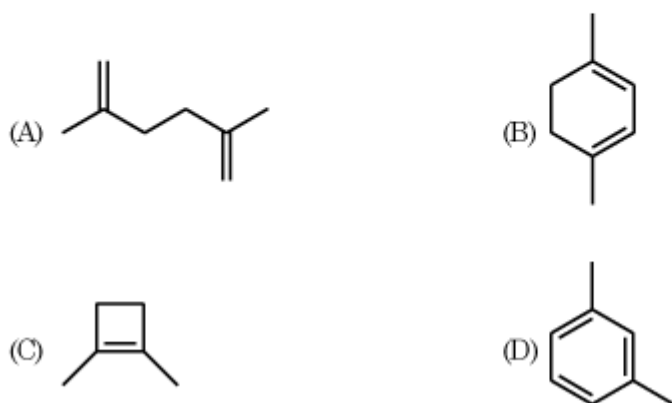
SECTION-II (More than one Correct Choice Type)

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24. Which of the following options is **CORRECT** ?
 (A) Tailing of mercury related to test of Ozone.
 (B) Carbon suboxide (C_3O_2) is linear molecule and prepared by dehydration of Malonic acid
 (C) K_2O and K_2O_2 are diamagnetic but KO_2 and KO_3 are paramagnetic
 (D) Mn_2O_7 is acidic oxide but MnO is amphoteric

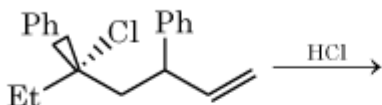


An organic compound on reaction with O_3 followed by Zn and H_2O gives P. The organic compound can be ?

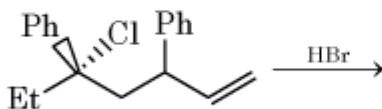


26. Select correct statement for following reactions :

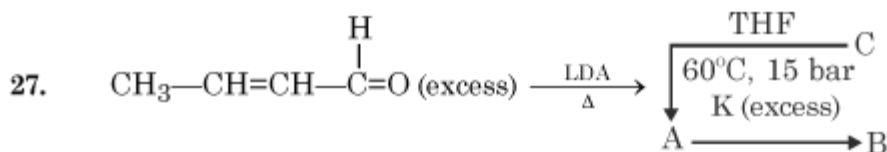
Reaction : I



Reaction : II

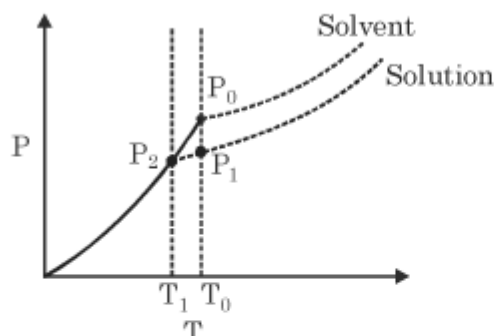


- (A) Reaction I gives mixture of 2 enantiomers & Reaction II gives mixture of two diastereomers.
 (B) Both reactions give mixture of 2 enantiomers.
 (C) Both reactions give mixture of 2 diastereomers.
 (D) Both reactions give 2 fraction of organic compounds.



- (A) 'A' is anti-aromatic.
 (B) 'B' is aromatic.
 (C) 'A' is having degree of unsaturation = 5.
 (D) In the compound 'C', C-H bond length is more than C-C bond length.

28. Find **CORRECT** statements.
- (A) Diamond is better conductor of heat than graphite.
 (B) Diamond is better conductor of electricity than Ag.
 (C) Diamond is electrical insulator.
 (D) Diamond gets converted to graphite at very high pressure & temperature.
29. From the phase diagram of water and an aqueous solution containing non volatile solute, identify the **correct** options :

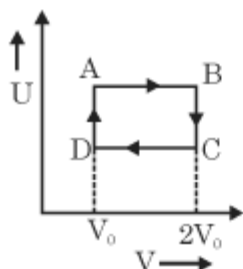


- (A) At temperature T₀, vapour pressure of solid solvent and vapour pressure of liquid solvent will be same.
 (B) Order of vapour pressure (P₀, P₁, P₂) are P₀ > P₁ > P₂.
 (C) $P_0 = P_2 e^{\frac{\Delta H_{\text{fusion}} [T_0 - T_1]}{R T_0 T_1}}$
 (D) $P_1 = P_2 e^{\frac{\Delta H_{\text{vap}} [T_0 - T_1]}{R T_0 T_1}}$
30. Which of the following statements are correct ?
- (A) $\frac{k_p}{k_a} = e^{\Delta E/RT}$ where k_p = Rate constant for reaction with catalyst.
 K_a = Rate constant for reaction without catalyst
 ΔE = Difference in activation energy for the two ways of reactions.
 (B) Alcohol acts as a negative catalyst for the conversion of
 $\text{Na}_2\text{SO}_3 \longrightarrow \text{Na}_2\text{SO}_4$.
 (C) TEL (tetraethyl lead) acts as a negative catalyst in the knocking of petrol.
 (D) Dilute H₂SO₄ acts as a positive catalyst in the hydrolysis of esters to carboxylic acids and alcohols.
31. In which of the following cases, the synergic bonding takes place at the π*-orbital of the ligand.
- (A) [PtCl₃(C₂H₄)]⁻
 (B) [Ni(PF₃)₄]
 (C) [Cr(C₆H₆)₂]
 (D) [Fe(π-C₅H₅)₂]

SECTION-III (Integer Type)

This section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

32. The internal energy (U) of an ideal (10.7 mole) gas is plotted against volume for a cyclic process ABCDA, as shown in the figure.



The temperature of the gas at B and C are 500 K and 300 K, respectively. The heat absorbed by the gas (in kcal) in this cyclic process is :

33. E_{Cell}° for the cell

$\text{Pt(s)} | \text{Fe}^{2+}(\text{aq}) | \text{Fe}^{3+}(\text{aq}) || \text{MnO}_4^{-}(\text{aq}), \text{H}^{+}(\text{aq}) | \text{Mn}^{2+}(\text{aq}) | \text{Pt}$ is 0.788 Volt

Given :

$$\Delta G_f^{\circ} \text{H}_2\text{O}(\ell) = -230 \text{ kJ mol}^{-1}$$

$$\Delta G_f^{\circ} \text{Fe}^{3+}(\text{aq}) = -10.2 \text{ kJ mol}^{-1}$$

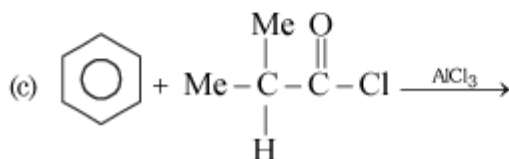
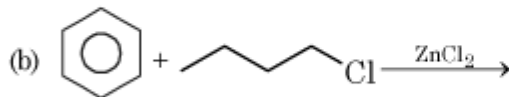
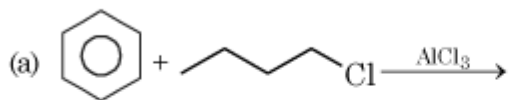
$$\Delta G_f^{\circ} \text{Mn}^{2+}(\text{aq}) = -229 \text{ kJ mol}^{-1}$$

$$\Delta G_f^{\circ} \text{Fe}^{2+}(\text{aq}) = -84 \text{ kJ mol}^{-1}$$

Magnitude of $\Delta G_f^{\circ} \text{MnO}_4^{-}(\text{aq})$ in kJ mol^{-1} is $x \times 10^2$ then 'x' is :

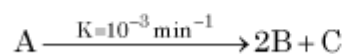
(Give your answer as nearest integer)

34. Three reactions are given below, write sum of total number of hydrogen atoms on all the carbon atoms which are connected directly by a single bond to benzylic carbon (carbon connected to benzene ring) in the products.



35. (a) From Meridional and facial isomer of $[Ma_3b_3]^{n\pm}$ on replacement of only one 'a' by 'b', the number of isomer of the product obtained are respectively x and y.
 (b) The number of corner shared per tetrahedron for 2D-silicate is z.
 Then $x + y + z$ is :

36. An optically active substance 'A' is decomposing into optically active substance 'B' and 'C' as



The specific rotations of A, B and C are $+40^\circ$, $+10^\circ$ and -30° per mole, respectively. Initially 'A' and 'C' were present in 4 : 3 mole ratio. Close to how many hours the sample becomes optically inactive ?

[Given : $\ln \frac{7}{5} = 0.34$, $\ln \frac{20}{13} = 0.42$]

PART-III : MATHEMATICS

SECTION-I (Single Correct Choice Type)

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

37. Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \min.(x + 2, 4 - 2x, 1 + 4x)$. The maximum value of $f(x)$ is equal to :

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{8}{3}$

38. Number of real roots of the equation

$$\sqrt{1+3x} + \sqrt{3+5x} + \sqrt{5+x} = \sqrt{3-x} + \sqrt{5-3x} + \sqrt{1-5x} \text{ is :}$$

(A) 1

(B) 2

(C) 3

(D) infinite

39. In a sequence $\{a_n\}$, $a_1 = 2$, $a_{n+1} = 1 - \frac{1}{a_n}$ for $n \geq 1$. Let P_n be the product of its first n terms, then the value of P_{2017} is

(A) $-\frac{1}{2}$

(B) 2

(C) $\frac{1}{2}$

(D) 1

40. Given that 'a' is a root of the equation $x^2 - x - 3 = 0$ then $\frac{a^3 + 1}{a^5 - a^4 - a^3 + a^2}$ equals

(A) $\frac{4}{3}$

(B) $\frac{3}{4}$

(C) $\frac{4}{9}$

(D) $\frac{9}{4}$

41. If $p, q, r \in \mathbb{R}$ satisfy $4p + 3q + r = 7$, then the least value of $(2p^2 + q^2 + r^2)$ is equal to :

(A) $\sqrt{7}$

(B) $\frac{7}{\sqrt{2}}$

(C) $\frac{\sqrt{7}}{2}$

(D) $\frac{49}{18}$

SECTION-II (More than one Correct Choice Type)

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42. Let us define system of vectors $\vec{u}_1 = \vec{v}_1$ and $\vec{u}_k = \vec{v}_k - \text{proj}_{\vec{u}_1}(\vec{v}_k) - \dots - \text{proj}_{\vec{u}_{k-1}}(\vec{v}_k)$

$$(\vec{v}_k), k \geq 2, \text{ where } \text{proj}_{\vec{a}}(\vec{b}) = \frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2} \text{ and } \vec{v}_1 = 2\hat{i} + \hat{j} - \hat{k}, \vec{v}_2 = \hat{i} + \hat{j} + \hat{k},$$

$\vec{v}_3 = 3\hat{i} - 2\hat{j} - \hat{k}$. Which of the following is correct ?

- (A) $[\vec{u}_1 \vec{u}_2 \vec{u}_3] \neq 0$
 (B) $\vec{u}_2 \cdot \vec{v}_1 \neq 0$
 (C) $\vec{u}_3 \cdot \vec{v}_1 = 0$
 (D) $\vec{u}_3 \cdot \vec{v}_2 \neq 0$

43. For a 3×3 invertible matrix A satisfying the characteristic equation $A^3 + pA^2 + qA - rI_3 = 0$, which of the following is/are true [Here $\text{tr}(A)$ = trace of matrix A, $\det(A)$ =determinant value of matrix A]

(A) $p = -\text{tr}(A)$

(B) $q = \frac{(\text{tr}(A))^2 - \text{tr}(A^2)}{2}$

(C) $r = \det(A)$

(D) $A^{-1} = \frac{-2(A - \text{tr}(A)I_3)}{(\text{tr}(A))^2 - \text{tr}(A^2)}$

44. Let $L_1 : \frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $L_2 : \frac{x-2}{-4} = \frac{y+1}{1} = \frac{z-6}{1}$ be two lines in R^3 .

Which of the following statement(s) is(are) correct ?

- (A) L_1, L_2 are coplanar.
 (B) L_1, L_2 are skew lines.
 (C) shortest distance between L_1 and L_2 is 9.
 (D) $(\hat{i} - 4\hat{j} + 8\hat{k})$ is a vector perpendicular to both L_1 and L_2 .

45. If $(x+1)(x+2) \dots (x+99) = A_0 + A_1 \cdot x + A_2 \cdot x^2 + \dots + A_{99} \cdot x^{99}$ and

$$S = \frac{\sum_{k=1}^{99} (k \cdot A_k)}{100!} = \sum_{i=1}^{99} \left(\frac{1}{r_i} \right), (r_i < r_j, \forall i < j) \text{ then value of } \sum_{k=1}^{99} \frac{1}{\sqrt{(r_k - 1)}} \text{ is lying}$$

between

- (A) 15 and 19
 (B) 15 and 17
 (C) 14 and 17
 (D) 17 and 19

46. Volume of parallelopiped formed by planes $2x + 2y + z = 4$, $2x + 2y + z = 1$, $2x - y + z = 3$, $2x - y + z = 6$, $3x + 4y + 12z = -1$, $3x + 4y + 12z = 12$ is $\frac{p}{q}$ where p and q are positive coprime integers then :
- (A) $p + q = 20$
 (B) $p + q = 57$
 (C) $p - q = 6$
 (D) $p - q = 5$
47. A line divides a plane into 2 regions. Two lines divide the plane into maximum 4 regions. If L_n is the maximum number of regions divided by n lines then which of the following is/are true :
- (A) $L_{20} = 211$
 (B) $L_{10} = 56$
 (C) $L_{15} = 121$
 (D) $L_{25} = 326$
48. Let $A(z_1)$ be the point of intersection of curves $\arg(z - 2 + i) = \frac{3\pi}{4}$ and $\arg(z + i\sqrt{3}) = \frac{\pi}{3}$. $B(z_2)$ be the point on the curve $\arg(z + i\sqrt{3}) = \frac{\pi}{3}$ such that $|z_2 - 5|$ is minimum and $C(z_3)$ be the centre of circle $|z - 5| = 3$.
 [Note : $i^2 = -1$].
 Then which of the following statement(s) is (are) correct ?
- (A) Distance between $A(z_1)$ and $B(z_2)$ is equal to 3.
 (B) Distance between $A(z_1)$ and $B(z_2)$ is equal to 4.
 (C) Area of triangle ABC is equal to $3\sqrt{2}$.
 (D) Area of triangle ABC is equal to $2\sqrt{3}$.
49. Given that the real numbers s, t satisfy $19s^2 + 99s + 1 = 0$, $t^2 + 99t + 19 = 0$ and $st \neq 1$. The absolute value of $\frac{st + 4s + 1}{t}$, is
- (A) a prime number
 (B) divisible by 19
 (C) divisible by 5
 (D) divisible by 2

SECTION-III (Integer Type)

This section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

50. Consider the system of equations $ax + y = b$, $bx + y = a$, $ax + by = ab$, where $a, b \in \{0, 1, 2, 3, 4\}$. The number of ordered pairs (a, b) for which system is consistent is equal to
51. Let $A = \begin{bmatrix} 4 & a \\ -1 & b \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ be two matrices satisfying the relation, $A^3 + 3A^2B + 3AB^2 + B^3 = (A + B)^3$. If $(A + B) = \lambda(A^{-1} + B^{-1})$, then λ is equal to
52. Let the focus of conic $\frac{(y+x)^2}{16} - \frac{(y-x)^2}{8} = 1$ is at (a, b) then $\frac{a^2 + b^2}{4}$ is

53. Let z and ω be complex numbers such that $z + \omega = i$ and $z^2 + \omega^2 = 1$. If the area of triangle formed by z , ω and origin is A , then $2\sqrt{80A^2 + 1}$ is equal to

54. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be non-constant differentiable function and satisfies,

$$f(x) = x^2 - \int_0^1 (x + f(t))^2 dt . \text{ Then } f(4) \text{ is equal to}$$