# SR,LT-IIT-MATHS-DIFFERENTIAL CALCULUS-DPP-06-2020-03-30

## SINGLE CORRECT ANSWER TYPE

01. Which of the following limit exists finitely ?

({.} denotes fractional part & [.] denotes G.I.F)

A) $\operatorname*{Lt}_{x\to 0}\dfrac{\{\tan x+\sin x\}}{\sin 2x}$

B) $\operatorname*{Lt}_{x\to 0}\dfrac{[\tan x+\sin x]}{\sin 2x}$

C) $\operatorname*{Lt}_{x\to 0}\dfrac{e^{6x}-1-3\{|\sin x|+|\tan x|\}}{x^2}$

D) $\operatorname*{Lt}_{x\to 0}\dfrac{(e^{6x}-1)[|x|]+\left\{\dfrac{\sin x}{x}\right\}}{[-|x|]}$

02. A function $f:R\to R$ is defined as $f(x)=\lim\limits_{n\to\infty}\dfrac{ax^2+bx+c+e^{nx}}{1+c\,e^{nx}}$, where '$f$' is continuous on R, then

A) $c=1, b\neq 0, a\in R$

B) $a=0, b\in R, c\neq 1$

C) $c=1, a,b\in R$

D) $c\neq 1, a,b\in R$

03. Let $g(x)=\ln f(x)$ where $f(x)$ is twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=x\,f(x)$ then for $N=1,2,3,\ldots\ldots;\ g''\left(N+\tfrac{1}{2}\right)-g''\left(\tfrac{1}{2}\right)=\ldots..$

A) $-4\left(1+\dfrac{1}{9}+\dfrac{1}{25}+\ldots\ldots+\dfrac{1}{(2N-1)^2}\right)$

B) $4\left(1+\dfrac{1}{9}+\dfrac{1}{25}+\ldots\ldots+\dfrac{1}{(2N-1)^2}\right)$

C) $-4\left(1+\dfrac{1}{9}+\dfrac{1}{25}+\ldots\ldots+\dfrac{1}{(2N+1)^2}\right)$

D) $4\left(1+\dfrac{1}{9}+\dfrac{1}{25}+\ldots\ldots+\dfrac{1}{(2N+1)^2}\right)$

04. A quadratic polynomial maps from $[-2,3]$ onto $[0,3]$ and touches X –axis at $x=3$, then the polynomial is

A) $\dfrac{3}{16}(x^2-6x+16)$ B) $\dfrac{3}{25}(x^2-6x+9)$ C) $\dfrac{3}{25}(x^2-6x+16)$ D) $\dfrac{3}{16}(x^2-6x+9)$

05. If $f(x)=x^3+bx^2+cx+d$ and $b^2<c$, then for all $x\in R$

(A) f(x) is strictly increasing function

(B) f(x) has a local maxima

(C) f(x) is strictly decreasing function

(D) cannot conclude monotonicity of f(x)

## MULTIPLE CORRECT ANSWER TYPE

06. Let $\lim\limits_{x\to 0}\left(2^x+a^x+e^x\right)^{1/x}=L$ which of the following statement(s) is (are) correct?

(A) If $L=a\left(a>0\right)$, then the range of a is $\left[e,\infty\right)$

(B) If $L=2e\left(a>0\right)$, then the range of a is $\left\{2e\right\}$

(C) If $L=e\left(a>0\right)$, then the range of a is $\left[0,e\right)$

(D) If $L=2a\left(a>1\right)$, then the range of a is $\left(\frac{e}{2},\infty\right)$

07. Let $f\left(x\right)=\begin{cases} x\left[\frac{1}{x}\right]+x\left[x\right] & if\ x\neq 0 \\ 0 & if\ x=0 \end{cases}$ (where $\left[x\right]$ denotes the greatest integer function).

Then the correct statement(s) is/are

A) Limit of $f\left(x\right)$ exists for $x=-1$

B) $f\left(x\right)$ has discontinuity at $x=1$

C) $f\left(x\right)$ has discontinuity at $x=2$

D) $f\left(x\right)$ is discontinuous at all positive integers

08. If $f\left(x\right)=\underset{n\to\infty}{Lt}\left(\sin x\right)^{2n}$ then which is/are correct for f(x)

A) Discontinuous at $x=\frac{\pi}{2}$

B) continuous at $x=\frac{\pi}{2}$

C) Discontinuous at $x=\frac{-\pi}{2}$

D) discontinuous at infinite number of points

09. If $f\left(x\right)=\begin{cases} 1-x & x\leq 0 \\ -\left(x-1\right)^2 & x\geq 1 \end{cases}$, the possible values of x for which $f\left(x\right)=f^{-1}\left(x\right)$ is

A) – 1 B) 0 C) 1 D) 2

10. For the curve represented parametrically by the equation, $x=2\log\left(\cot t\right)+1$ and $y=\tan t+\cot t$, then

A) Tangent at $t=\frac{\pi}{4}$ is parallel to X – axis

B) Normal at $t=\frac{\pi}{4}$ is parallel to Y – axis

C) Tangent at $t=\frac{\pi}{4}$ is parallel to $y=x$

D) Normal at $t=\frac{\pi}{4}$ is parallel to $y=x$

**COMPREHENSION TYPE**

**Paragraph for Questions 11 to 12**

Let $f(x)=\lim\limits_{n\to\infty}\dfrac{x^2+2(x+1)^{2n}}{(x+1)^{2n+1}+x^2+1}, n\in N$ and $g(x)=\tan\left(\dfrac{1}{2}\sin^{-1}\left\{\dfrac{2f(x)}{1+f^2(x)}\right\}\right)$

11. If $x\in(-2,0)$ then the range of $g(x)$ is a subset of

(A) $\left(0,\dfrac{4}{5}\right)$ (B) $\left(\dfrac{4}{5},\infty\right)$ (C) $\left(0,\dfrac{5}{4}\right)$ (D) $\left(\dfrac{5}{4},\infty\right)$

12. $\lim\limits_{x\to-3}\dfrac{\sin(x+3)g(x)}{x^2+4x+3}$ is equal to

(A) $0$ (B) $-2$ (C) $\dfrac{1}{2}$ (D) Non-existent

**Paragraph for Question Nos. 13& 14**

Let $f:R\longrightarrow R$ be a function defined as

$f(x)=\begin{cases}1-|x| & ;|x|\le 1\\ 0 & ;|x|>1\end{cases}$ and $g(x)=f(x-1)+f(x+1)$, $\forall x\in R$ then

13. The number of points at which $g(x)$ is discontinuous is/are

A) 0 B) 1 C) 2 D) 4

14. The number of points at which $g(x)$ is non-differentiable is/are

A) 3 B) 4 C) 5 D) 6

## Paragraph for Question Nos. 15 to 16.

Let $f:N\cup\{0\}\to R$ be a function such that $f(0)=0$, $f(1)=1$ and $f(n+1)=4f(n)-f(n-1)$ for $n=1,2,3...$

15. $f(n)$ is

A) $\dfrac{1}{2\sqrt{3}}\left(\left(2+\sqrt{3}\right)^n-\left(2-\sqrt{3}\right)^n\right)$ B) $\dfrac{1}{2\sqrt{3}}\left(\left(2-\sqrt{3}\right)^n+\left(2+\sqrt{3}\right)^n\right)$

C) $\dfrac{1}{2\sqrt{3}}\left(\left(2-\sqrt{3}\right)^n-\left(2+\sqrt{3}\right)^n\right)$ D) None of these

16. For $n = 1,2,3...$, $f^2(n) - f(n-1)\,f(n+1)$ is

A) 1 for $n \in \{1,2,3,4\}$ and 2 for $n \geq 5$ B) 1 for all, $n \in N$

C) $(-1)^n \ \forall n \geq 5$ D) None of these

**INTEGER TYPE**

17. Let $g(x)$ be a function defined on [0,1] such that $g(x) = \begin{cases} x \quad , & x \text{ is rational} \\ 1 - x, & x \text{ is irrational} \end{cases}$.

Given that $f(f(x)) = lx + m \quad \forall x \in [0,1]$ then the value of $l^2 + l + 2lm$ equals

18. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$=

19. Let g(x) = f(x – 1) + f(x + 1); where $f(x) = \begin{cases} 1 - |x|; & |x| \leq 1 \\ 0; & |x| > 1 \end{cases}$. The number of points of intersection with $y = \sin\alpha$; (where $\alpha$ is constant and $0 < \alpha < \pi$)

20. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is

21. Let f be a function which is continuous and differentiable for all real x. If f(2) = – 4 and $f'(x) \geq 6$ for all $x \in [2,4]$, then find least value of $f(4)$

## MATCHING TYPE

22. Match the following from column-I to column-II

| | Column – I (Function) | | Column – II (Properties) |
|---|---|---|---|
| P) | $x\|x\|$ | 1) | Continuous on $(-1,1)$ |
| Q) | $\sqrt{\|x\|}$ | 2) | Differentiable on $(-1,1)$ |
| R) | $x+[x]$ where $[.]$ is G. I. F | 3) | Strictlyincreasing in $(-1,1)$ |
| S) | $\|x-1\|+\|x+1\|$ | 4) | Not differentiable for a point in $(-1,1)$ |

| | P | Q | R | S |
|---|---|---|---|---|
| **A)** | 1,2 | 1,4 | 3,4 | 1,2 |
| **B)** | 2,3 | 3,4 | 1,4 | 1,2 |
| **C)** | 1,2,3 | 1,4 | 3,4 | 1,2 |
| **D)** | 1,2,3 | 1,4 | 1,2 | 3,4 |

23. MATCH THE FOLLOWING

| | Column – I | | Column – II |
|---|---|---|---|
| A) | f(x)=$(x-1)^3(x-2)^5$ has/is | P) | Local maxima |
| B) | $f(x)=\begin{cases}\sin\left(\frac{\pi x}{4}\right) & ;x\le 2\\ 9-4x & ;x>2\end{cases}$ is/has in $(0,3)$ | Q) | Local minima |
| C) | $f(x)=\{2x\}$ ({.} denotes fractional part of $x$) is/has in $(0,1)$ | R) | Continuous |
| D) | $f(x)=\begin{cases}\|\|x-2\|-2\| & ;x<2\\ [x] & ;x\ge 2\end{cases}$ (where [.] = G.I.F), then $f(x)$ is/has in $(-1,4)$ | S) | Non - differentiable |
| | | T) | differentiable |

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|---|---|---|---|---|---|
| **01.D** | **02.C** | **03.A** | **04.B** | **05.A** | **06.ABC** |
| **07.ABCD** | **08.A** | **09.D** | **10.AB** | **11.AC** | **12.C** |
| **13.A** | **14.C** | **15.A** | **16.B** | **17.2** | **18.1** |
| **19.4** | **20.8** | **21.8** | **22.C** | **23.A** | |